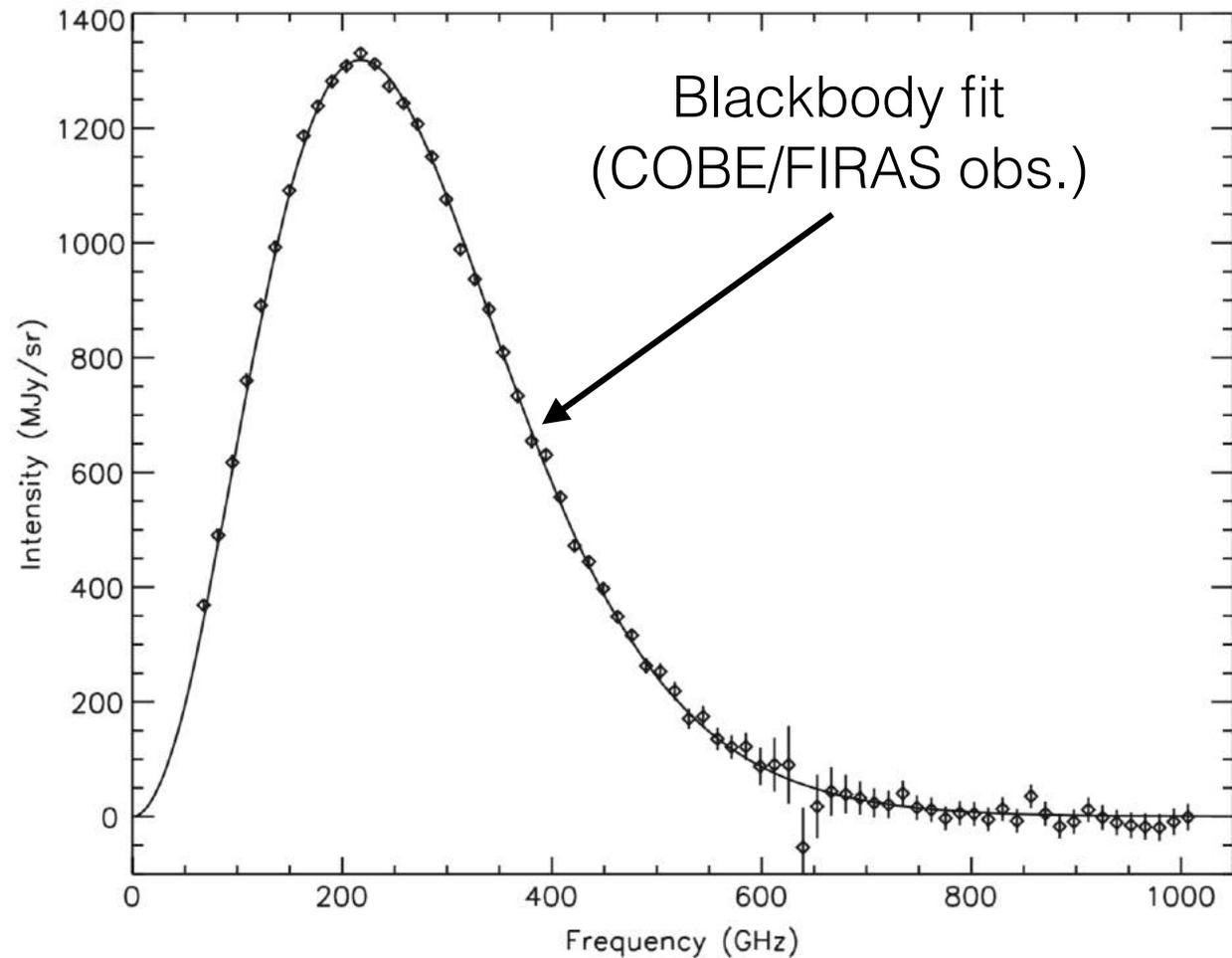


Basic observational properties of the CMB

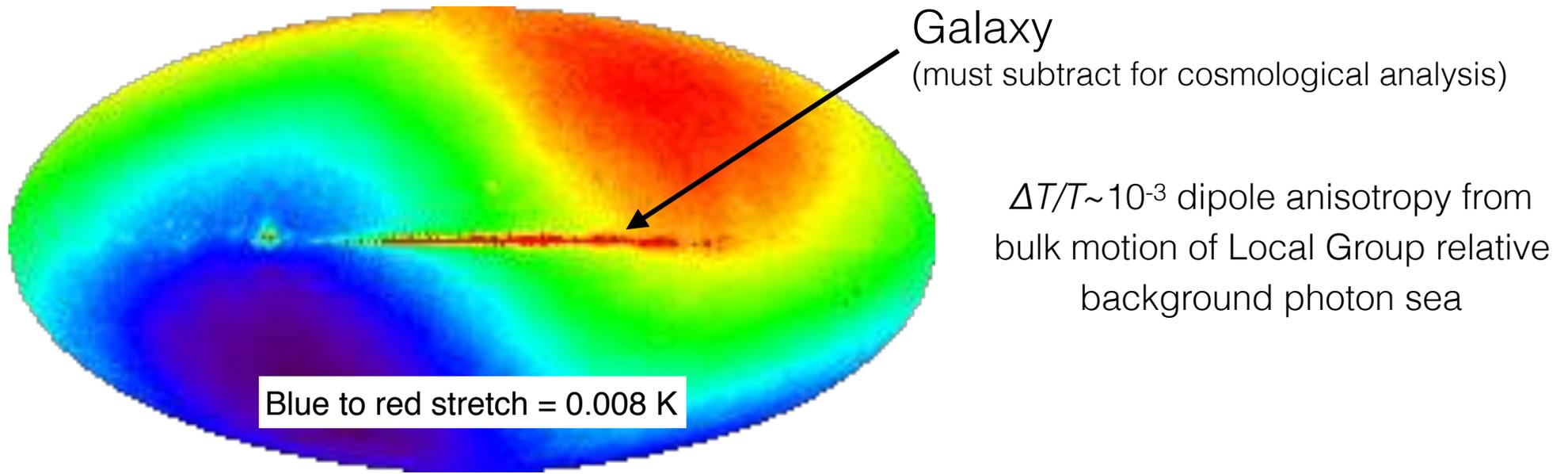
Mean CMB spectrum

At any angular position (θ, ϕ)
on sky, BB with
 $T = 2.7256 \pm 0.0006$ K

Deviations from perfect BB
 $\Delta T/T \lesssim 10^{-4}$



CMB dipole



$$\frac{\Delta T}{T} \approx \frac{v}{c} \approx 2 \times 10^{-3} \Rightarrow v_{\text{LG}} \approx 600 \text{ km/s}$$

velocity of Local Group relative to CMB

Note: dipole usually measured after correcting for other known motions:

- orbit of Earth around Sun (~ 30 km/s)
- orbit of Sun around center of Galaxy (~ 220 km/s)
- orbit of Galaxy relative to Local Group (~ 80 km/s)

CMB picks out a particular rest frame: the one in which observers see no dipole anisotropy. Comoving observers can be thought of being at rest w.r.t. to CMB.

Origin of dipole pattern

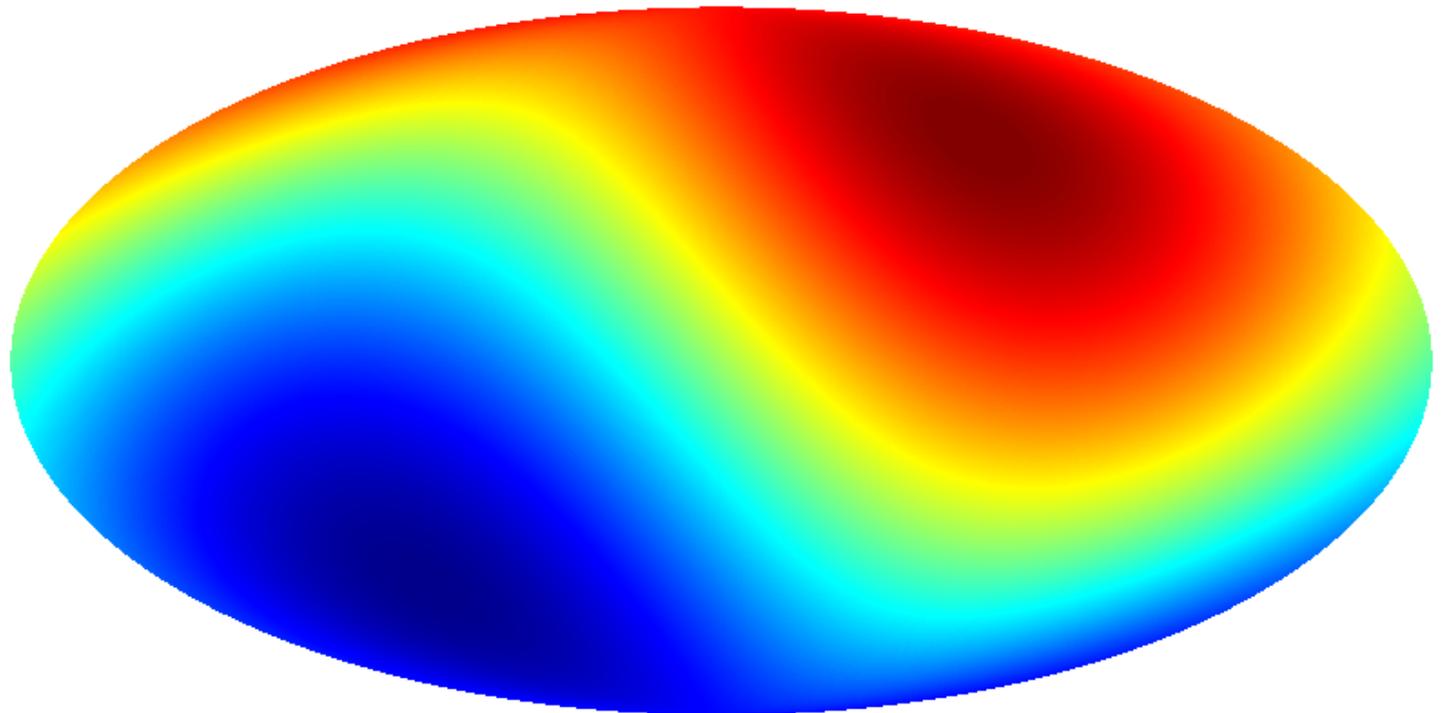
First-order expansion of
relativistic Doppler:

$$T(\theta) = T_0 \frac{(1 - \beta^2)^{1/2}}{1 - \beta \cos \theta} \approx T_0(1 + \beta \cos \theta)$$

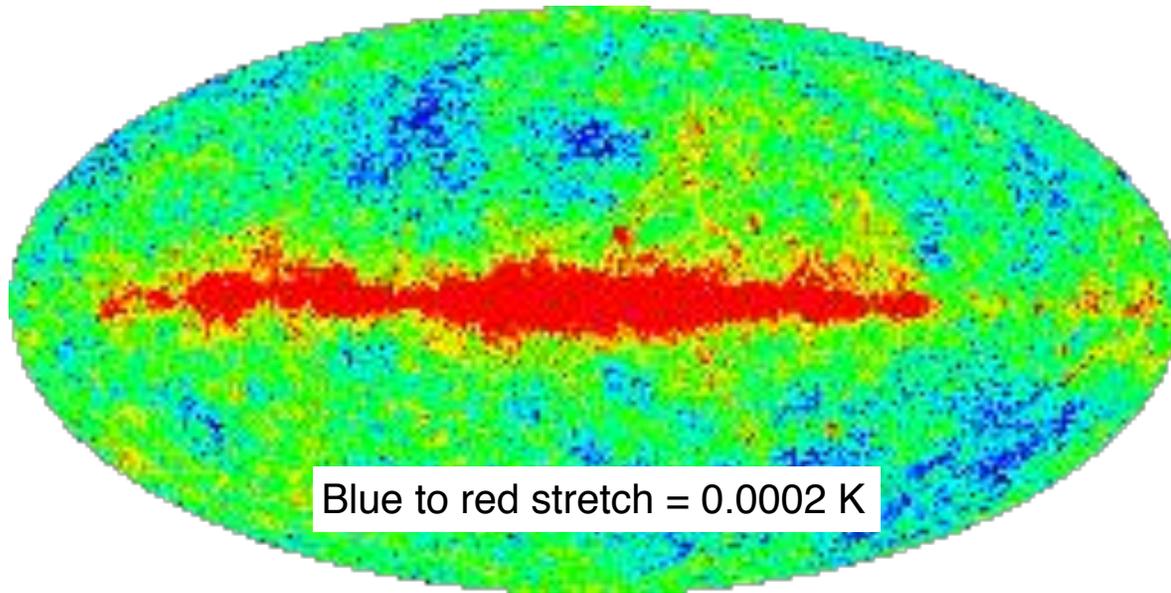
$$\beta = v/c$$

θ = angle of observation relative to \mathbf{v}

Hotter (higher T) for $\theta \sim 0$



(Residual) CMB anisotropies



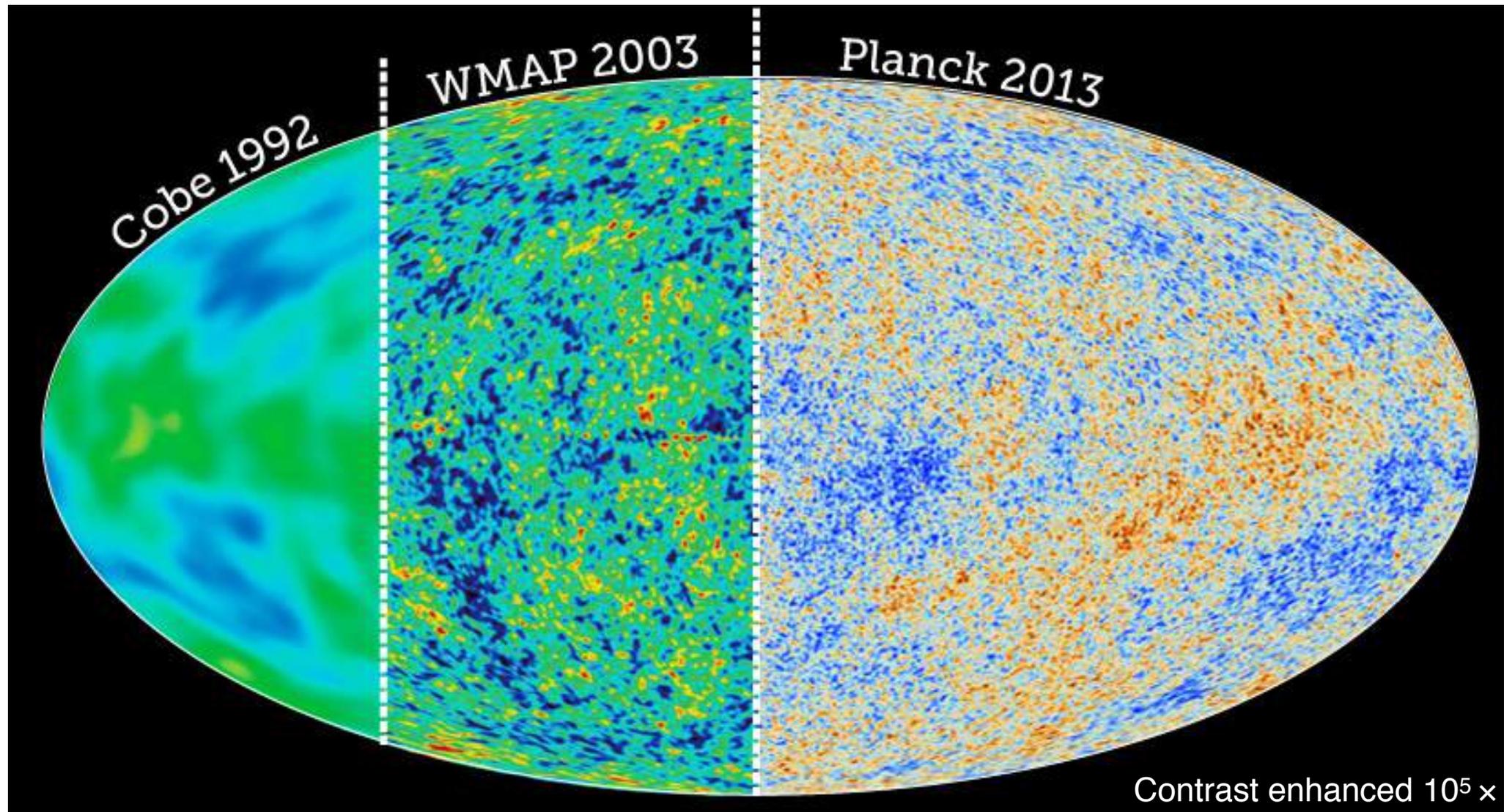
$\Delta T/T \sim 10^{-5}$ fluctuations
after subtracting dipole

$$\langle T \rangle = \frac{1}{4\pi} \int_0^{2\pi} \int_0^\pi d\phi d\theta \sin \theta T(\theta, \phi)$$

$$\frac{\delta T}{T}(\theta, \phi) = \frac{T(\theta, \phi) - \langle T \rangle}{\langle T \rangle}$$

$$\Rightarrow \left\langle \left(\frac{\delta T}{T} \right)^2 \right\rangle^{1/2} \sim 10^{-5}$$

Three satellites have provided full-sky maps of CMB anisotropies



There are also ground- and balloon-based CMB experiments. Ground-based provide the best angular resolution (diffraction limit $\Theta \sim \lambda/D$) but cannot map the whole sky.

Baryon-to-photon ratio (η)

Baryons: $n_{b,0} = \frac{\Omega_{b,0}\rho_{\text{crit},0}}{m_p} \approx 3 \times 10^{-7} \text{ cm}^{-3}$

($\Omega_{b,0} = 0.05$, $H_0 = 70 \text{ km/s/Mpc}$)

Photons: $\rho_{\text{rad},0} \equiv \frac{\epsilon_{\gamma,0}}{c^2}$ $\epsilon_{\gamma,0} = \alpha T_0^4$ $\alpha = \frac{\pi^2 k_B^4}{15\hbar^3 c^3}$

$$\Omega_{\text{rad},0} = \frac{\rho_{\text{rad},0}}{\rho_{\text{crit},0}} \approx 5 \times 10^{-5}$$

$$n_{\gamma,0} = 16\pi\zeta(3) \left(\frac{k_B T_0}{hc} \right)^3 \approx 420 \text{ cm}^{-3}$$

Ratio:

$$\Rightarrow \eta \equiv \frac{n_{b,0}}{n_{\gamma,0}} \approx 7 \times 10^{-10}$$

- ▶ η^{-1} very large, important to understand relatively low T at which recombination and BBN occur
- ▶ property of our universe, not well understood — why didn't anti-baryons annihilate all the baryons?

Cosmological recombination and last scattering

Saha correctly predicts z_{rec} but incorrect in detail

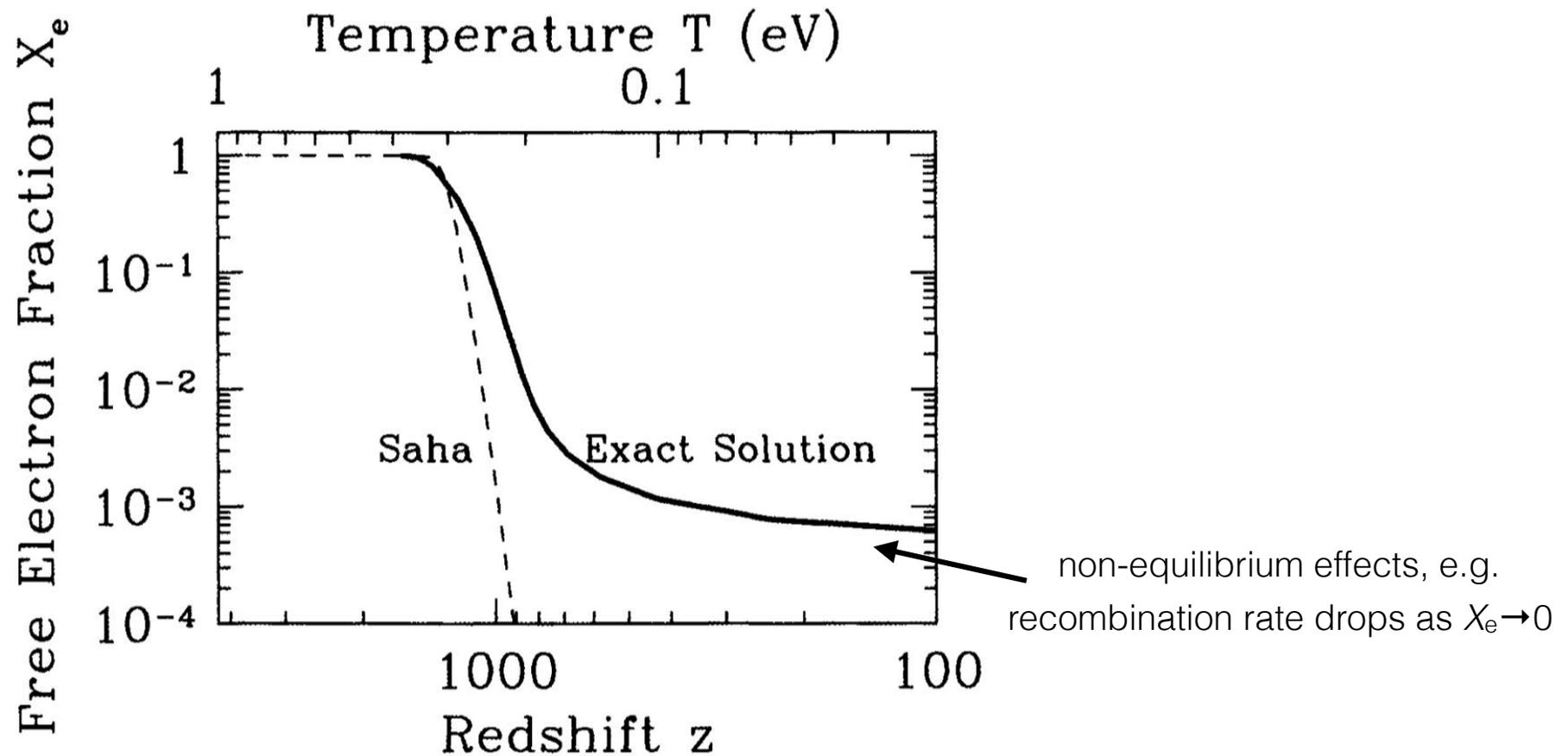


Figure 3.4. Free electron fraction as a function of redshift. Recombination takes place suddenly at $z \sim 1000$ corresponding to $T \sim 1/4$ eV. The Saha approximation, Eq. (3.37), holds in equilibrium and correctly identifies the redshift of recombination, but not the detailed evolution of X_e . Here $\Omega_b = 0.06$, $\Omega_m = 1$, $h = 0.5$.

The CMB power spectrum

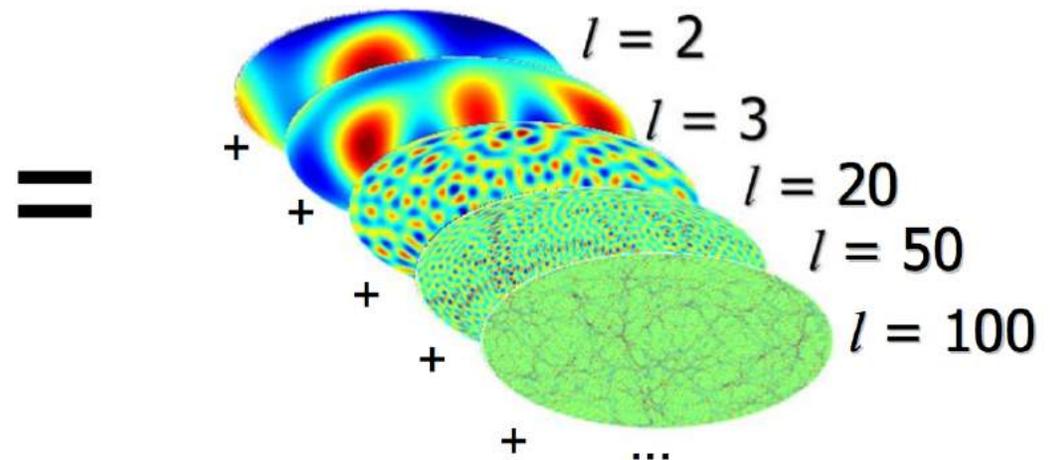
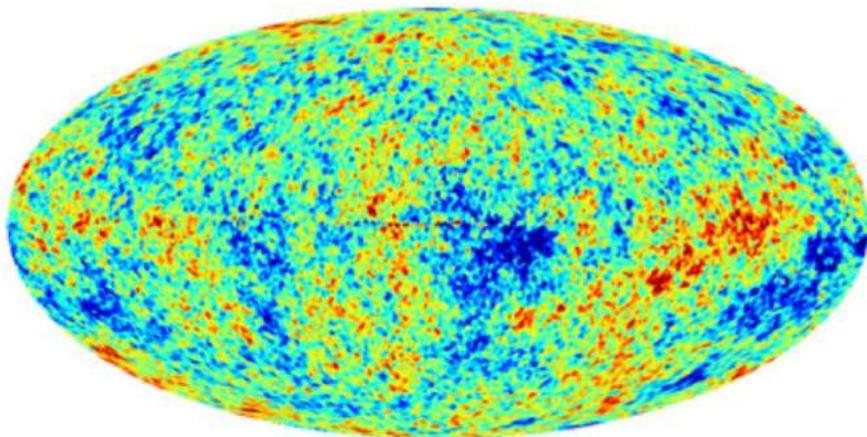
Spherical harmonics representation of the CMB sky

- ▶ Orthonormal basis so can use to represent any scalar field on the sky
- ▶ Spherical analog of Fourier decomposition
- ▶ Apply to CMB temperature

$$T(\hat{n}) = \sum_{\ell=0}^{\ell_{\max}} \sum_{m=-\ell}^{\ell} a_{\ell m} Y_{\ell m}(\hat{n})$$

$$a_{\ell m} = \int_{4\pi} T(\hat{n}) Y_{\ell m}^*(\hat{n}) d\Omega$$

$$\sqrt{\frac{2\ell + 1}{4\pi} \frac{(\ell - m)!}{(\ell + m)!}} P_{\ell m}(\cos \theta) e^{im\phi} \equiv Y_{\ell m}(\theta, \phi)$$



Angular power spectrum

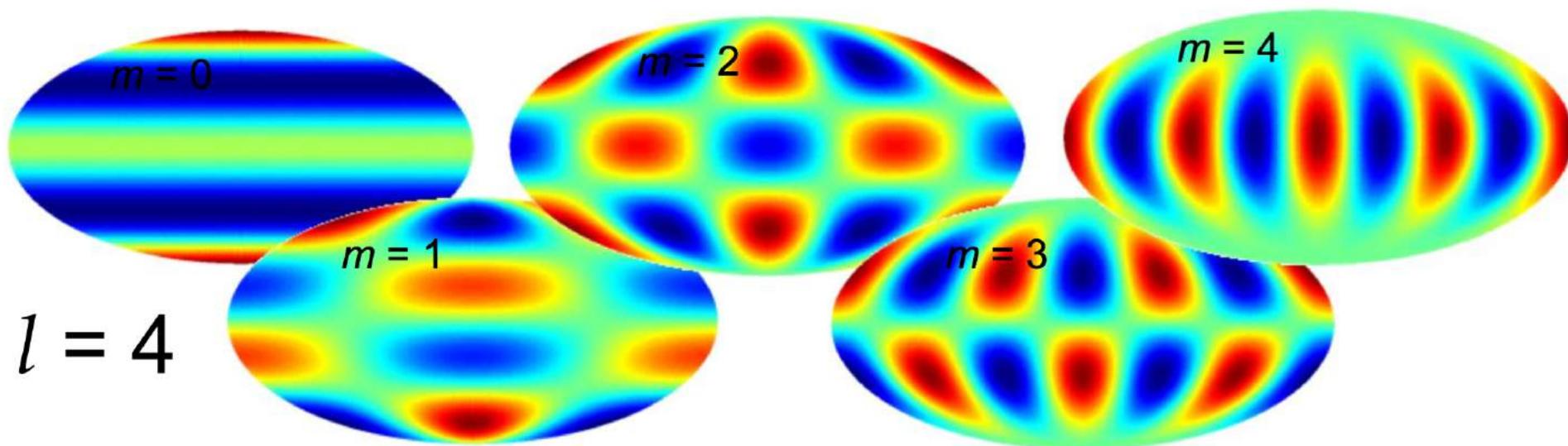
$$C_\ell = \frac{1}{2\ell + 1} \sum_{m=-\ell}^{\ell} |a_{\ell m}|^2$$

$$[a_{\ell m}] = \mu K \Rightarrow [C_\ell] = \mu K^2$$

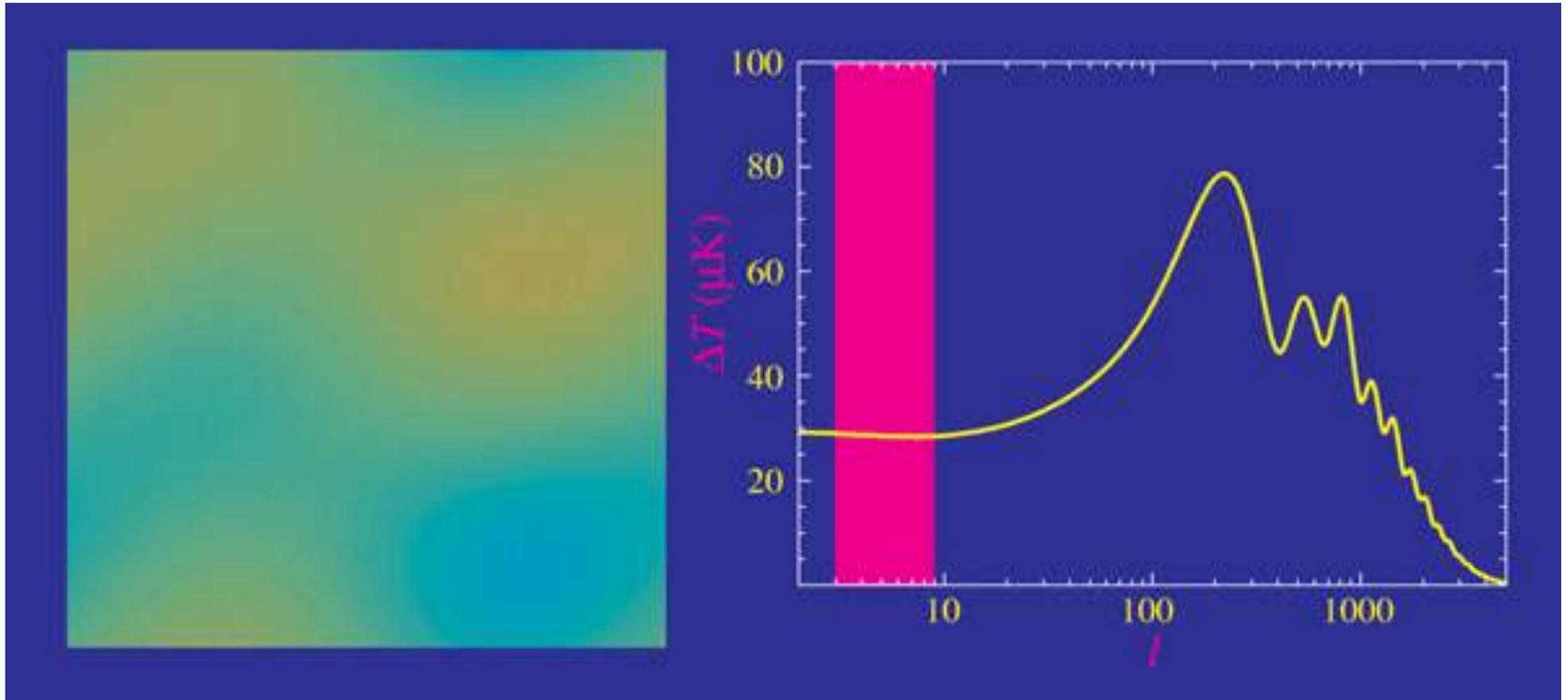
Often plotted is $D_\ell = \ell(\ell+1)C_\ell/2\pi$

► ℓ determines the wavelength of the mode (number of waves along meridian, so $\ell \sim 1/\theta$)

► m determines the “shape” of the mode (number of modes along equator)



Low l = large angular scales, high l = small scales



Magenta band on the right represents a filter on the angular size of features in the map on the left. The filter starts on the 10 degree scale similar to the original COBE measurements. As the filter passes through the first peak in the power spectrum, the spots are degrees in scale and most intense.

Modern CMB power spectrum: WMAP + high-res ground-based (SPT, ACT)

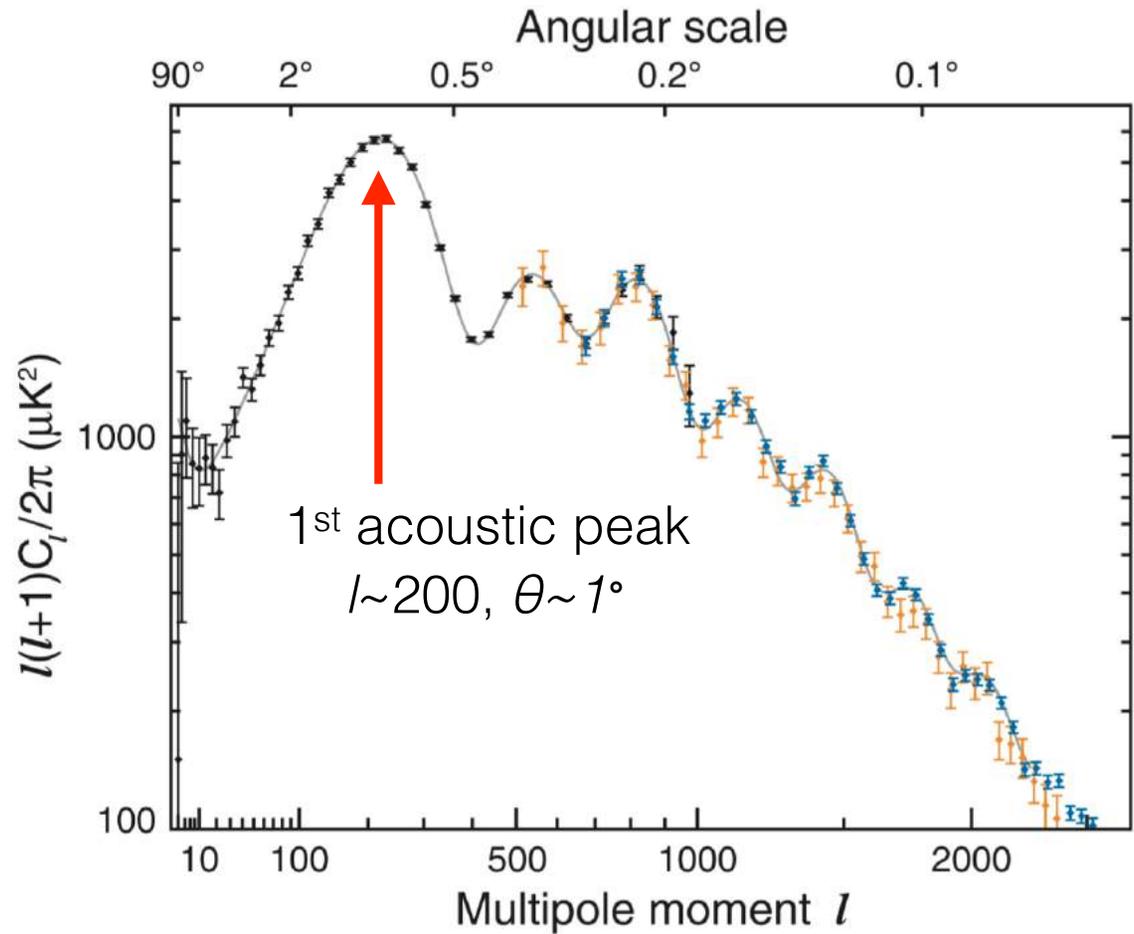


Figure 1. Compilation of the CMB data used in the nine-year *WMAP* analysis. The *WMAP* data are shown in black, the extended CMB data set—denoted “eCMB” throughout—includes SPT data in blue (Keisler et al. 2011) and ACT data in orange, (Das et al. 2011b). We also incorporate constraints from CMB lensing published by the SPT and ACT groups (not shown). The Λ CDM model fit to the *WMAP* data alone (shown in gray) successfully predicts the higher-resolution data.

Modern CMB power spectrum: Planck

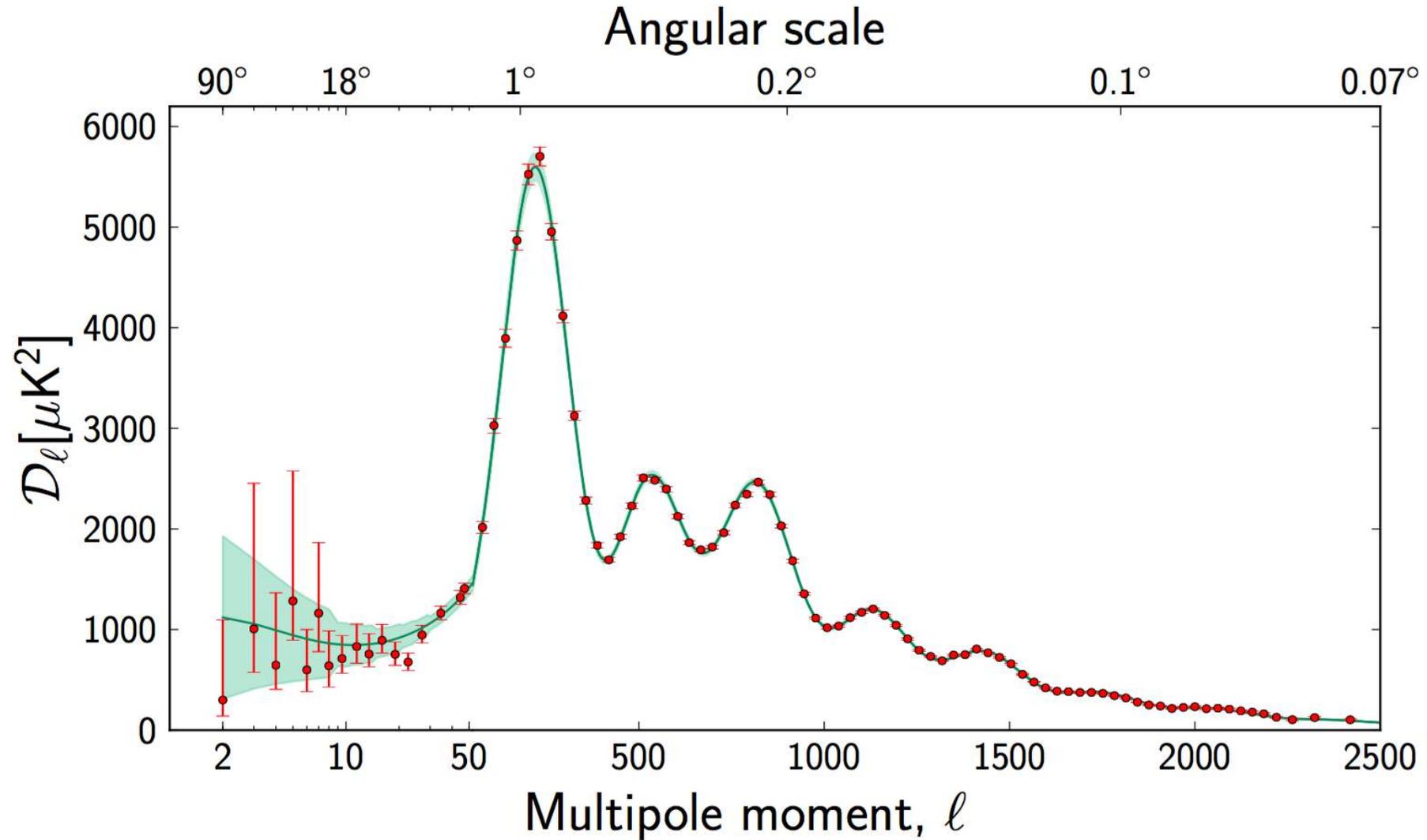


Figure 37. The 2013 *Planck* CMB temperature angular power spectrum. The error bars include cosmic variance, whose magnitude is indicated by the green shaded area around the best fit model. The low- ℓ values are plotted at 2, 3, 4, 5, 6, 7, 8, 9.5, 11.5, 13.5, 16, 19, 22.5, 27, 34.5, and 44.5.

What physical scales are probed by CMB anisotropies?

$$d_A(z_{\text{LS}} \approx 1, 100) \approx 12 \text{ Mpc}$$

$$\Rightarrow \delta l = d_A \delta\theta \approx 12 \text{ Mpc} \left(\frac{\delta\theta}{1 \text{ rad}} \right) \quad \text{proper size at last scattering}$$

$$\approx 0.22 \text{ Mpc} \left(\frac{\delta\theta}{1^\circ} \right)$$

$$\delta l(\text{today}) = (1 + z_{\text{LS}}) \delta l(z_{\text{LS}})$$

$$\approx 240 \text{ Mpc} \left(\frac{\delta\theta}{1^\circ} \right) \quad \text{first acoustic peak}$$

Must go to $\ll 1^\circ$ ($l \sim 200 \times 240 \sim 48,000$) to resolve ~ 1 Mpc galaxy clusters — not accessible to even highest resolution current CMB experiments

→ CMB fluctuations probe very large physical scales!

Physics of acoustic oscillations

Conditions at last scattering

$$\begin{aligned}\rho_{\text{DM}}(z_{\text{LS}}) &= \rho_{\text{crit},0} \Omega_{\text{DM}} (1 + z_{\text{LS}})^3 \\ &\approx 3.2 \times 10^{-21} \text{ g cm}^{-3}\end{aligned}$$

$$\Omega_{\text{DM}} = \Omega_{\text{m}} - \Omega_{\text{b}}$$

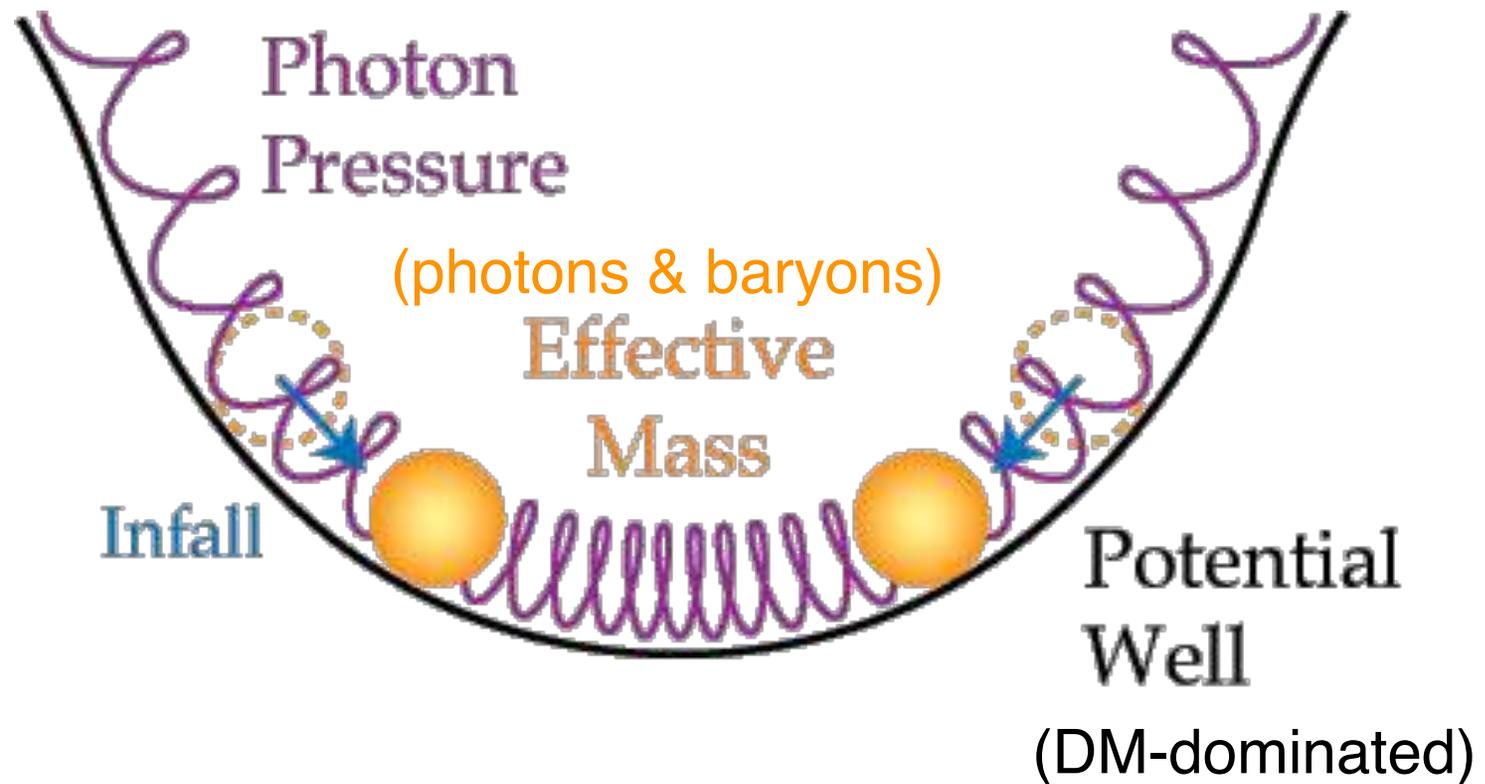
$$\rho_{\text{b}}(z_{\text{LS}}) \approx 5.0 \times 10^{-22} \text{ g cm}^{-3}$$

$$\begin{aligned}\rho_{\text{rad}}(z_{\text{LS}}) &= \rho_{\text{rad},0} (1 + z_{\text{LS}})^4 \\ &\approx 6.8 \times 10^{-22} \text{ g cm}^{-3}\end{aligned}$$

$$\Rightarrow \text{at last scattering: } \rho_{\text{DM}} : \rho_{\text{rad}} : \rho_{\text{b}} = 6.4 : 1.4 : 1$$

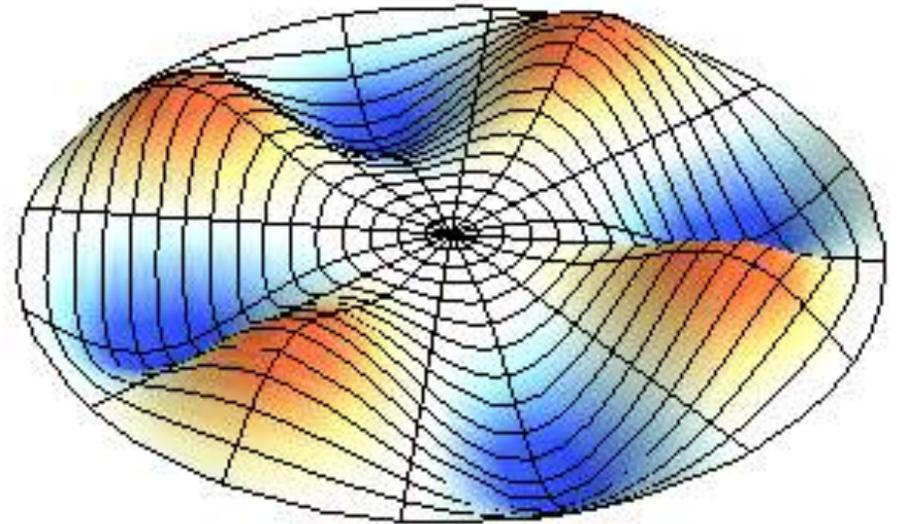
DM dominates potential but not electromagnetically coupled to baryon-photon plasma — DM perturbations not prevented from growing by pressure of the plasma (except indirectly via gravity)

Baryon-photon fluid falls into dark matter potential wells, then rebounds once pressure becomes sufficient



Sound waves in baryon-photon plasma

Think of pressure in plasma acting as a restoring force analogous to tension in a string or drum skin



Growing dark matter perturbations gravitationally “pluck” the plasma, launching sound waves traveling at speed

$$c_s \approx \left(\frac{dP}{d\rho} \right)^{1/2} \approx \frac{c}{\sqrt{3}}$$

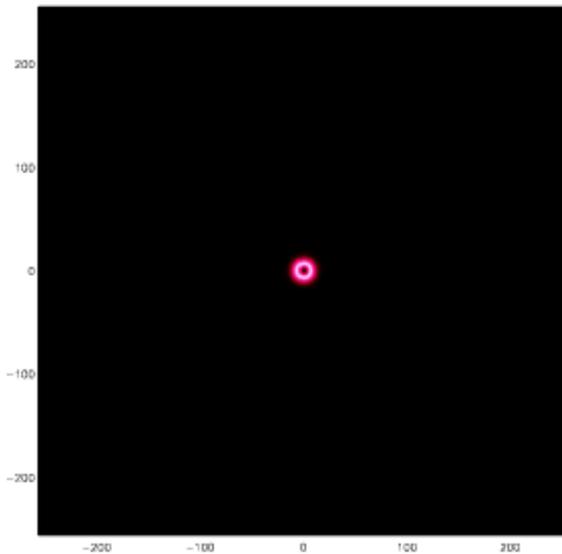
$$(P \approx \rho c^2 / 3)$$

(neglecting baryons
in this approx.)

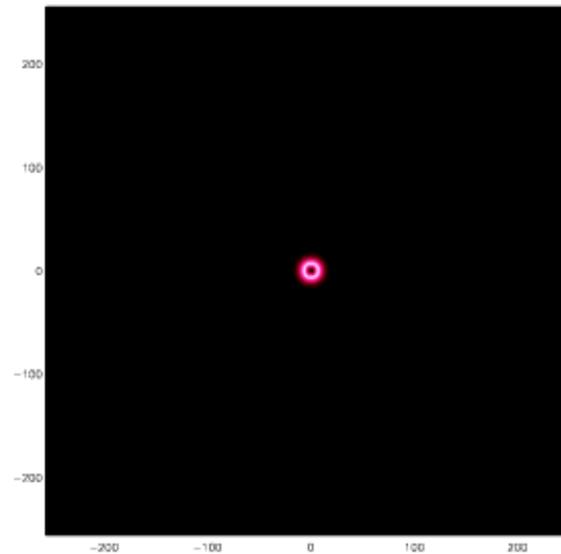
Animated illustrations of acoustic oscillations

Consider the early Universe, which was composed of a coupled plasma of photons, ionized hydrogen, and dark matter. Start with a single perturbation. In this example, the plasma is totally uniform except for an excess of matter at the origin. High pressure drives the gas+photon fluid outward at a speed $\sim c/3^{1/2}$

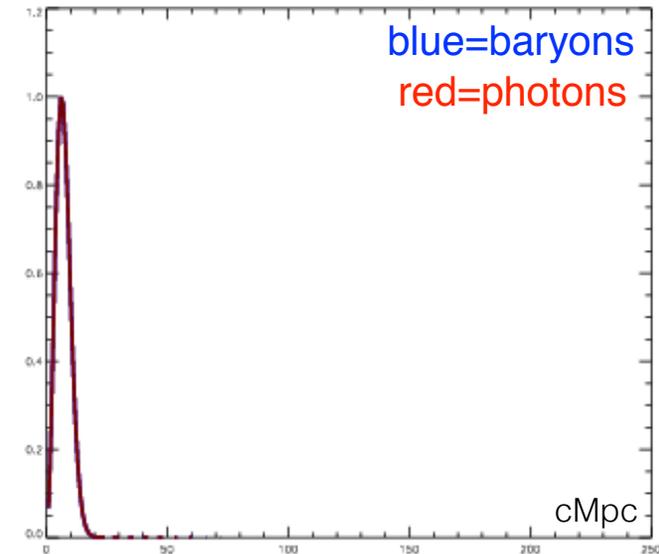
Baryon density



Photon density

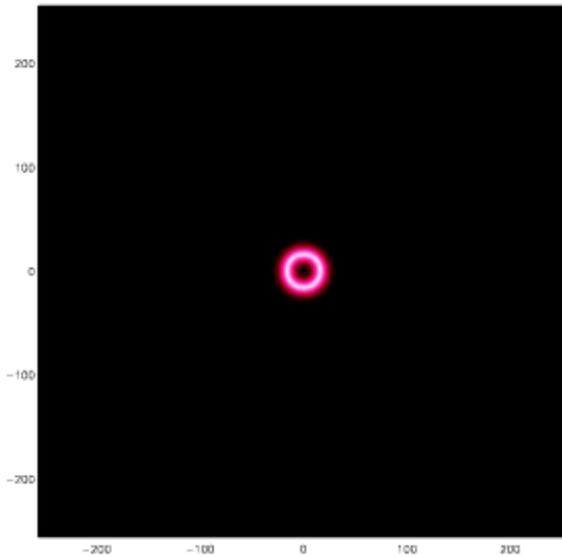


Radial mass profiles

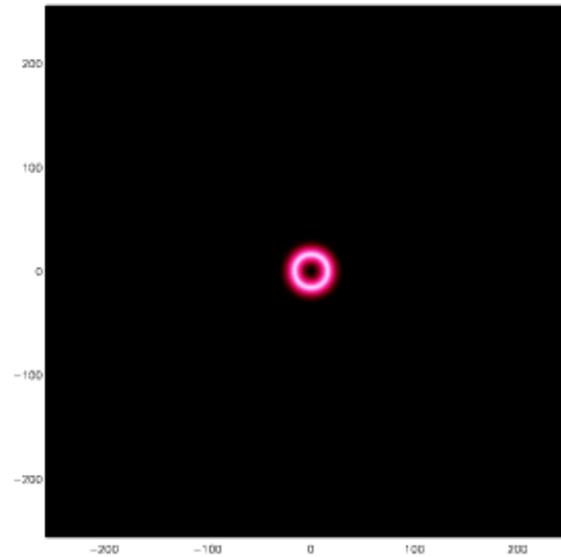


Initially (before decoupling) both the photons and the baryons move outward together.

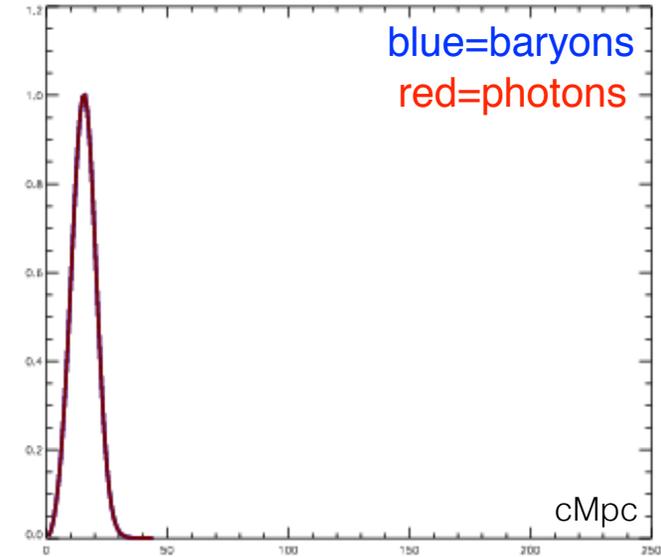
Baryon density



Photon density

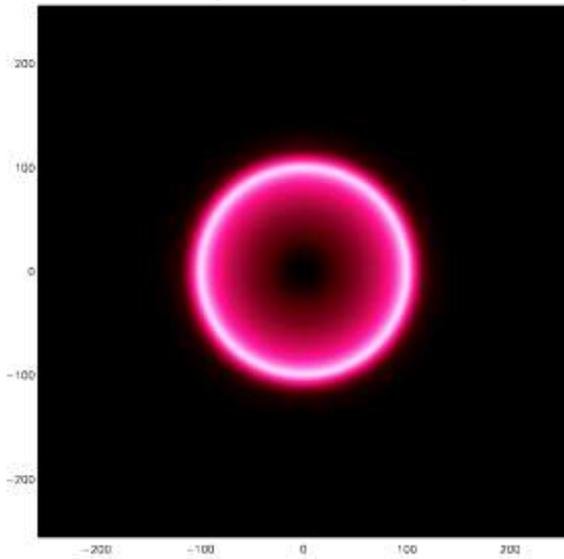


Radial mass profiles

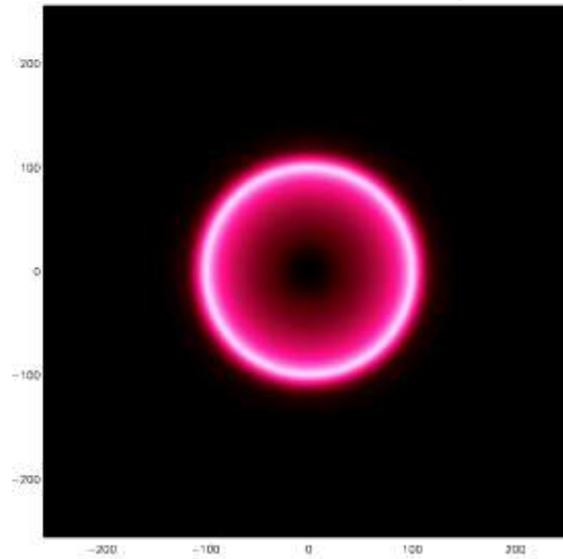


This expansion continues for $\sim 300,000$ years

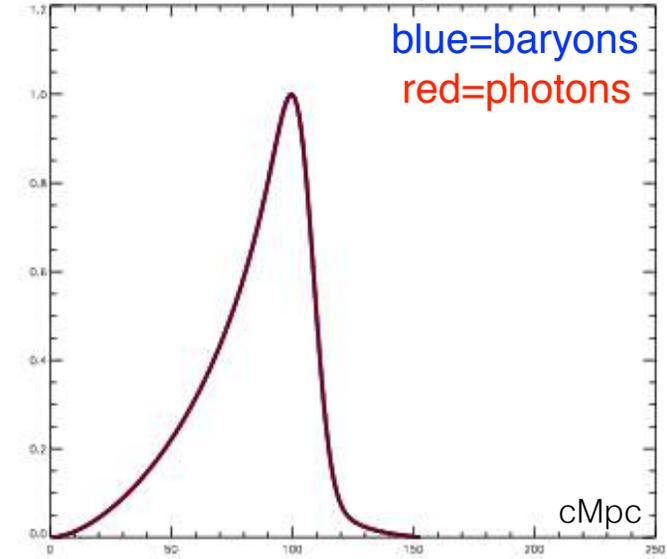
Baryon density



Photon density

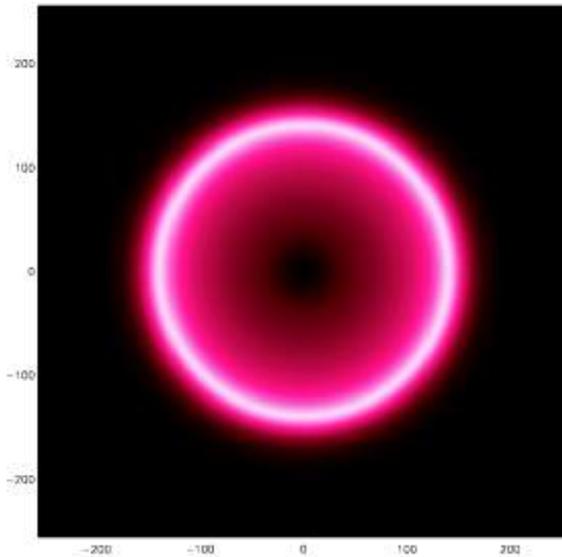


Radial mass profiles

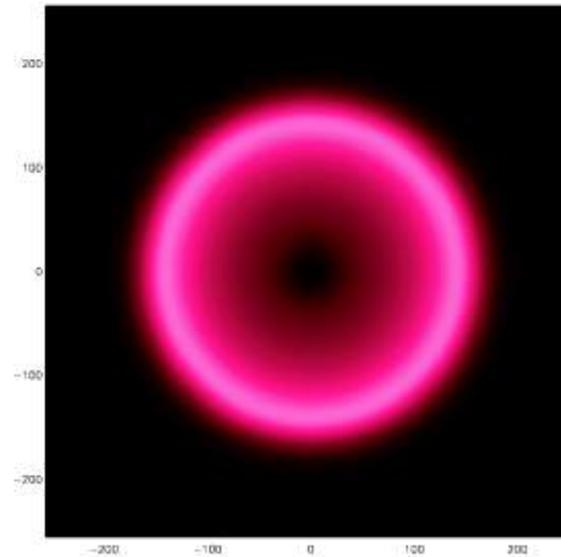


After $\sim 300,000$ years, the Universe has cooled enough the protons capture the electrons to form neutral hydrogen. This decouples the photons from the baryons. The former quickly stream away, leaving the baryon peak stalled.

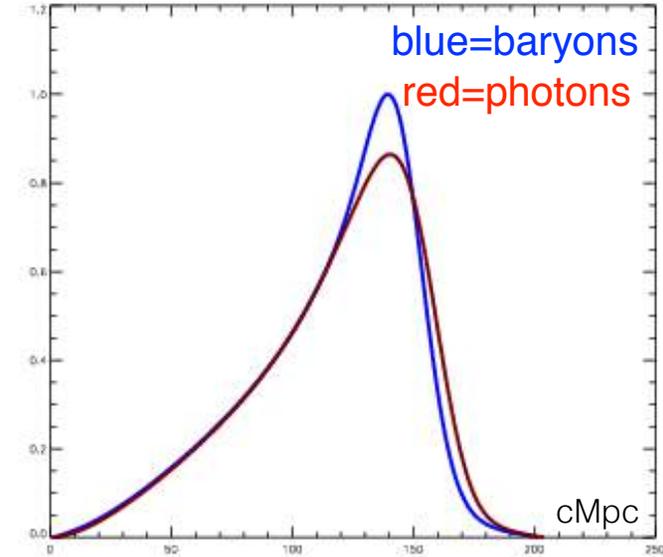
Baryon density



Photon density

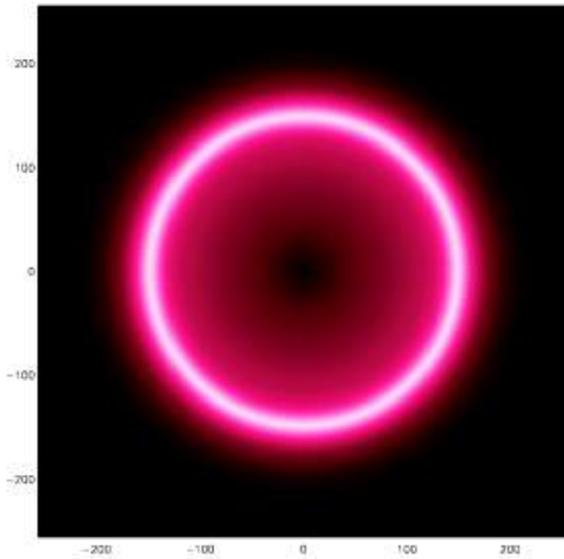


Radial mass profiles

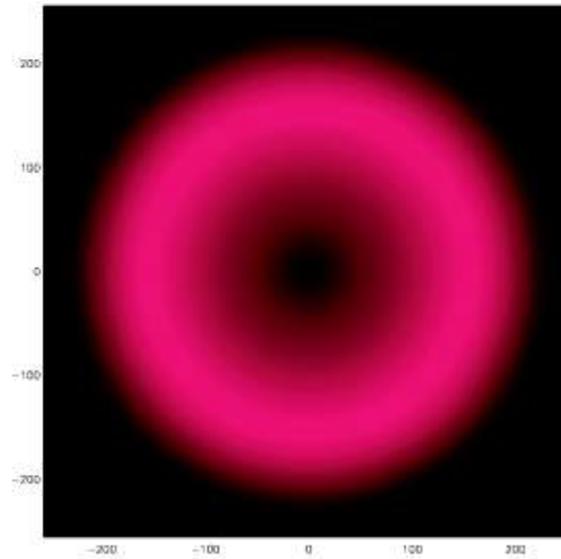


The photons continue to stream away while the baryons, having lost their motive pressure, remain in place.

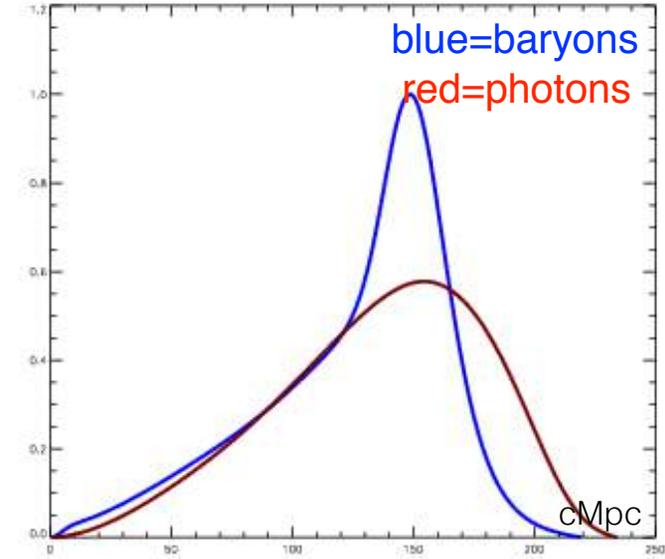
Baryon density



Photon density

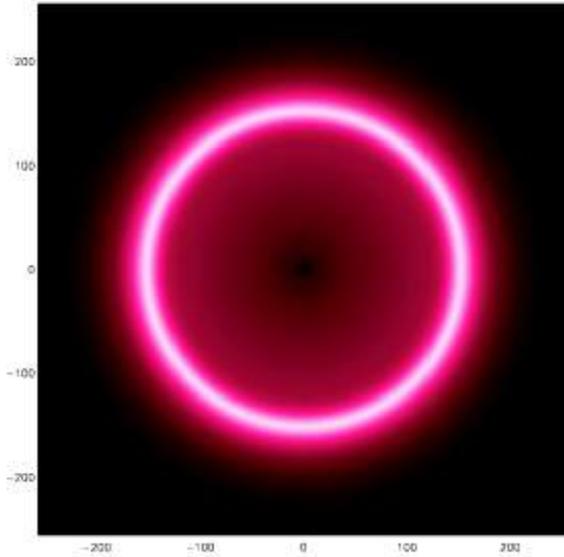


Radial mass profiles

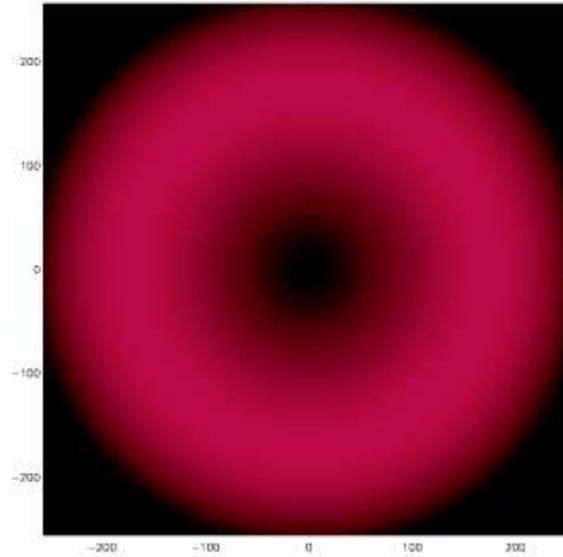


Decoupled evolution of photons and baryons continue for a while...

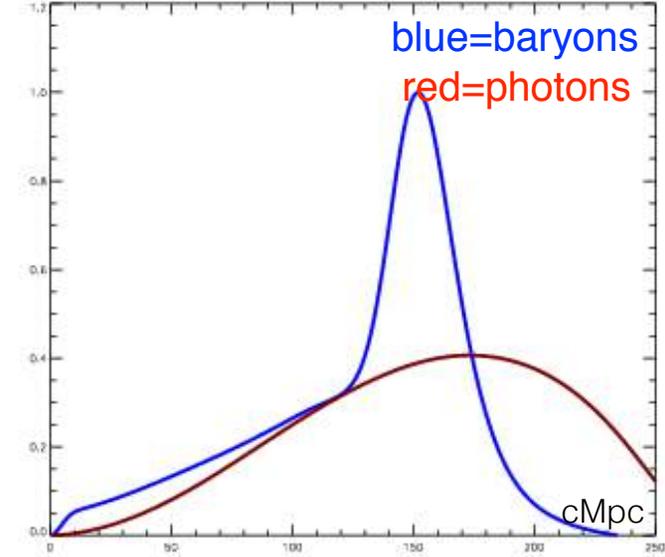
Baryon density



Photon density

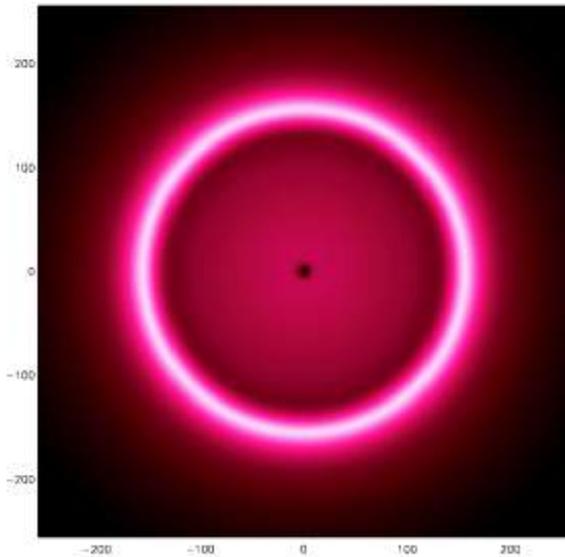


Radial mass profiles

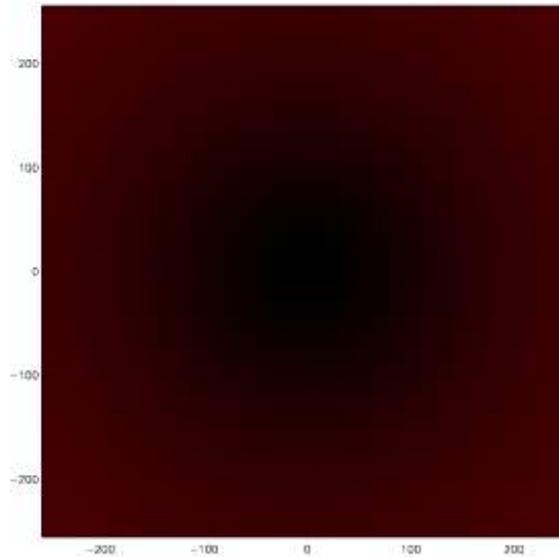


The photons have become almost completely uniform, but the baryons remain overdense in a shell ~ 150 Mpc (comoving) in radius. In addition, the large gravitational potential well which we started with starts to draw material back into it.

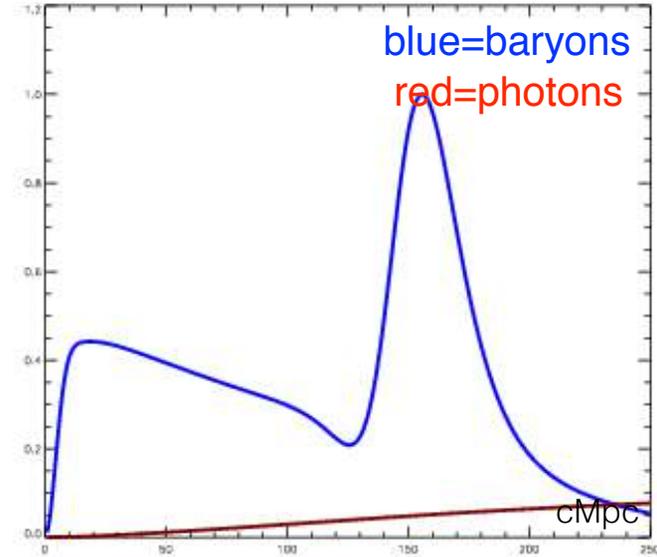
Baryon density



Photon density

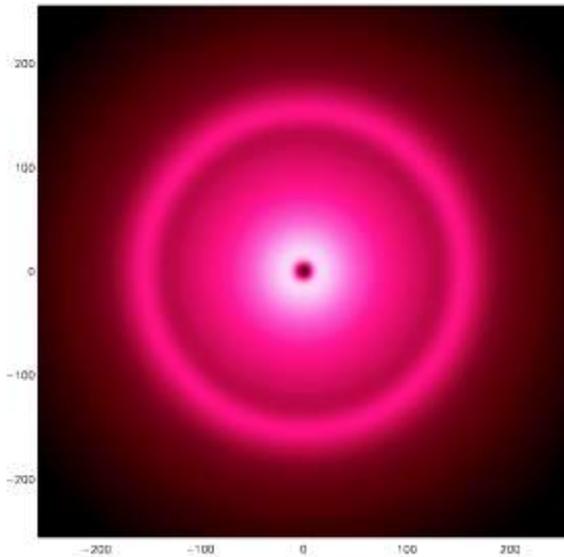


Radial mass profiles

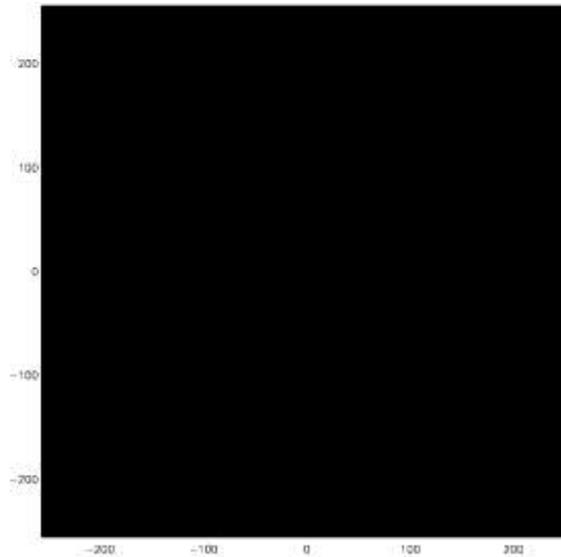


As the perturbation grows by $O(1,000)$ the baryons and DM reach equilibrium densities in the ratio Ω_b/Ω_m . The final configuration is our original peak at the center (which was put in by hand) and an echo in a shell ~ 150 Mpc in radius. The radius of this shell is known as the sound horizon.

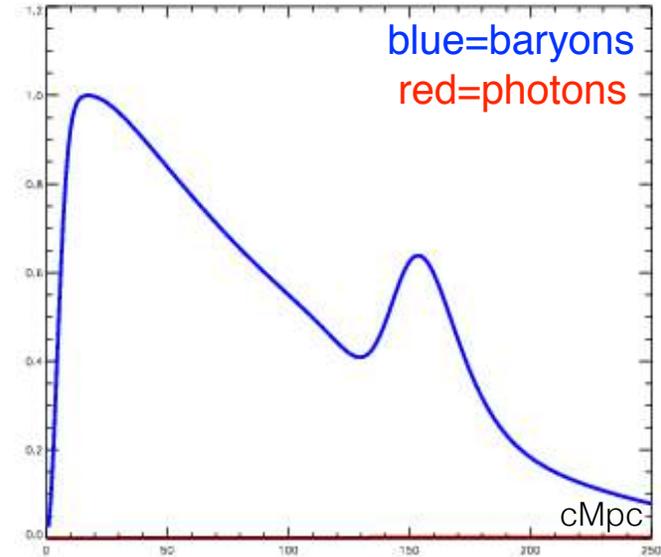
Baryon density



Photon density

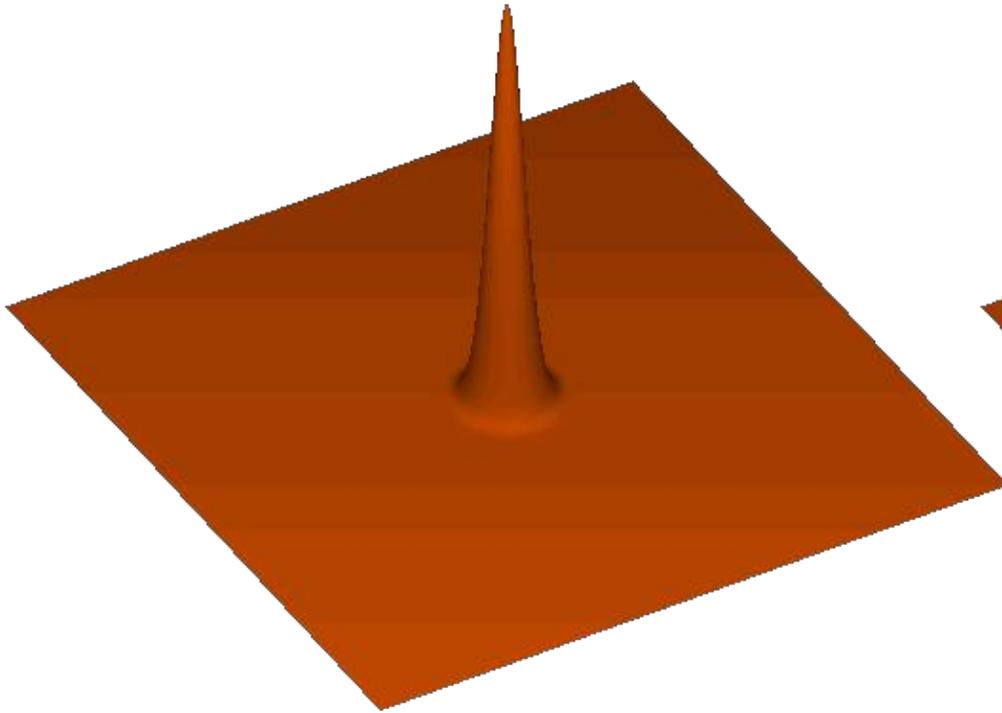


Radial mass profiles

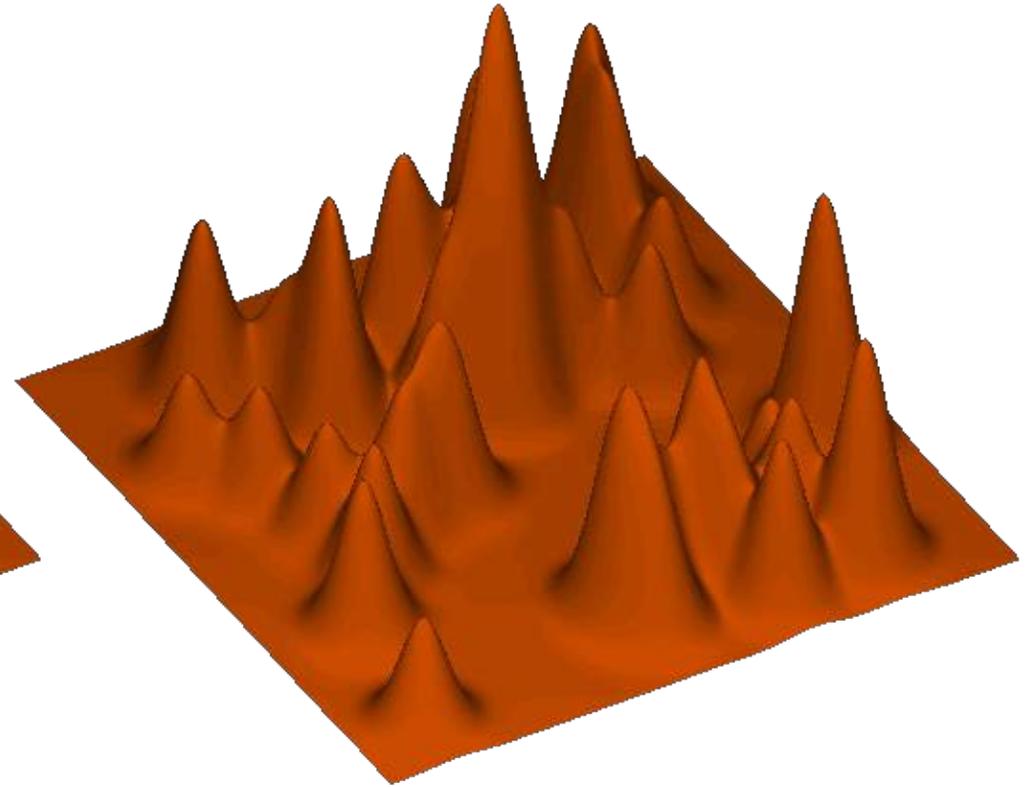


In reality, there is a spectrum of perturbations:
single vs. superposed waves in baryons

One wave



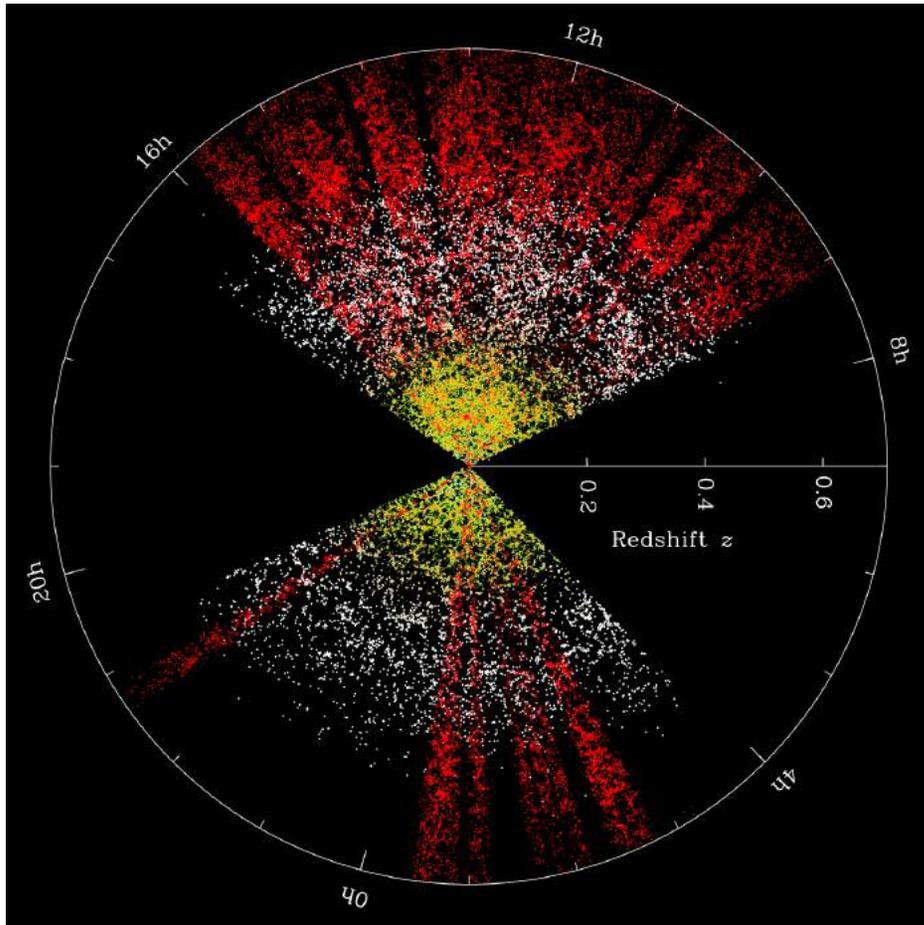
Multiple waves



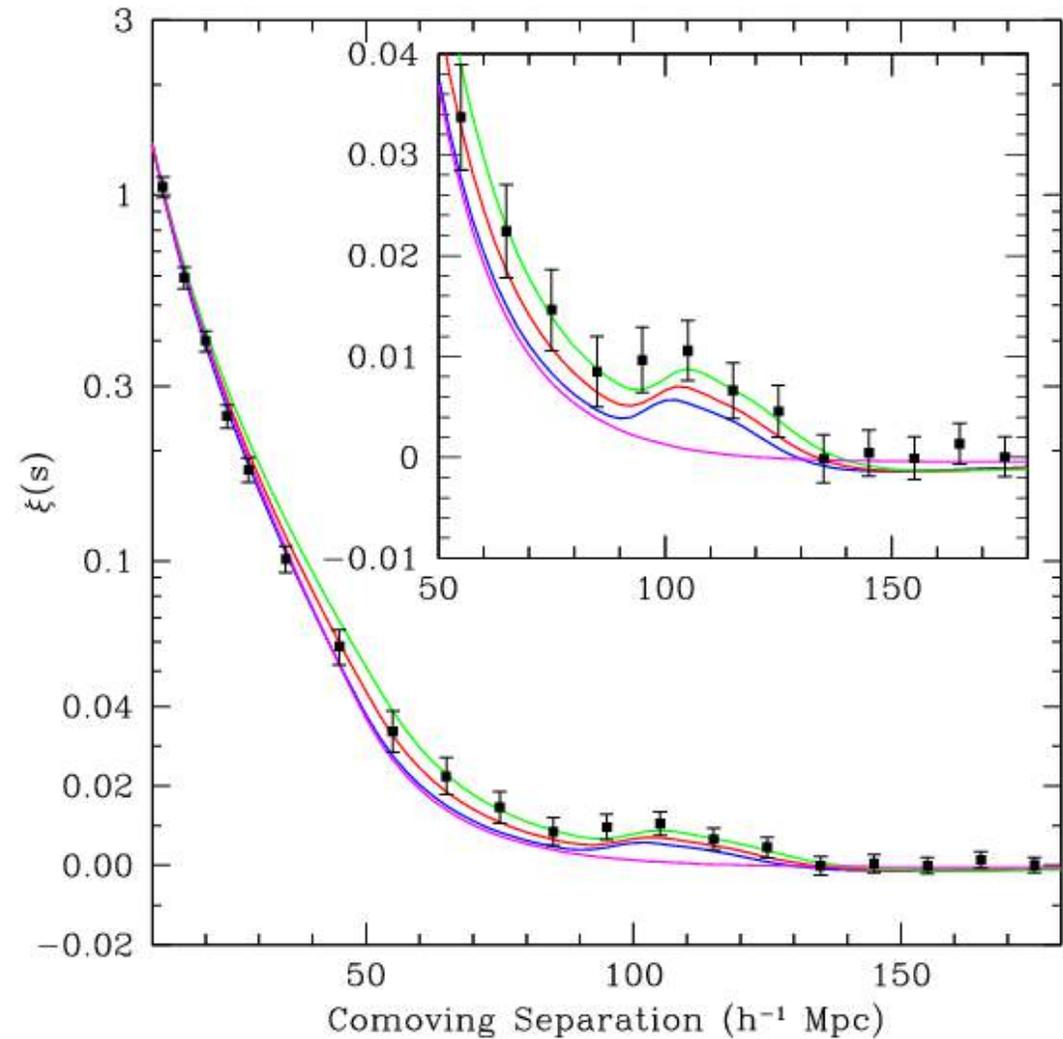
Color = transition
from ionized to
neutral

CMB fluctuations grow into “baryonic acoustic oscillations” in the galaxy distribution

Baryonic Oscillation Spectroscopy Survey (BOSS) galaxy map



Luminous red galaxy (LRG) correlation function



**Additional effects:
Sachs-Wolfe and Silk damping**

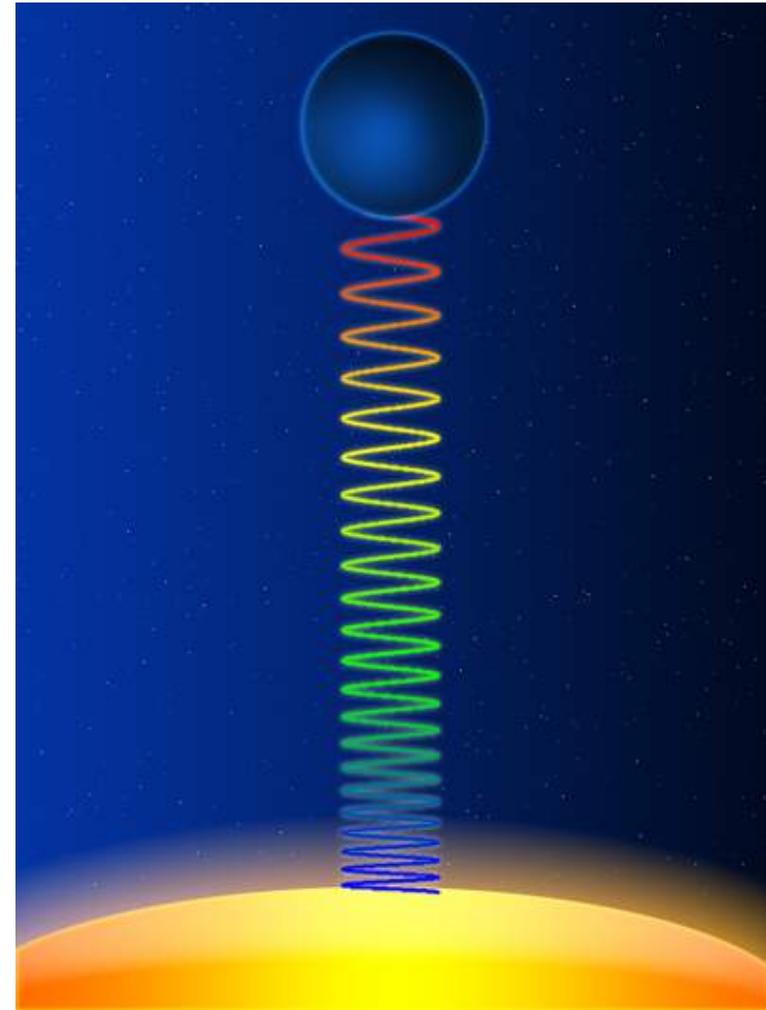
Gravitational redshift

Photons are redshifted when they climb out of potential wells. E.g.:

- from surface of a star to infinity
- from Earth's surface to the top of a building
- from a dark matter-dominated over-density in the early universe to outside of it

To first order,
$$\frac{\Delta\nu}{\nu} \approx -\frac{\Delta\phi}{c^2}$$

(climbing out $\Rightarrow \Delta\phi > 0$, $\Rightarrow \Delta\nu < 0$, i.e. redshift)



Pound-Rebka experiment (1959)

Gravitational redshift
measured from bottom
of Harvard's physics
building to its top
("Harvard tower"
experiment)



Sachs-Wolfe effect

As baryon-photon fluid falls into dark matter-dominated over-densities...

- it is compressed, so might expect photons coming from matter over-densities at LS to have $\Delta T/T > 0$ (in analogy with $T \propto a^{-1}$ as universe expands)
- Sachs & Wolfe (1969) showed that gravitational redshifting effect is larger and *matter over-densities* actually correspond to CMB *cold spots*

On the net:
$$\frac{\delta T}{T} = \frac{1}{3} \frac{\delta \Phi}{c^2}$$
 (local density enhancements correspond to $\delta \Phi < 0$)

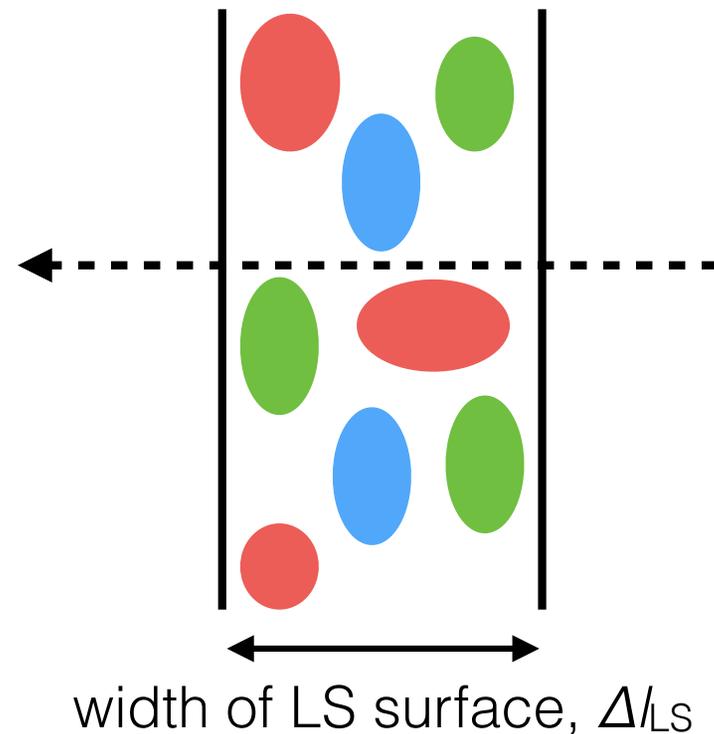
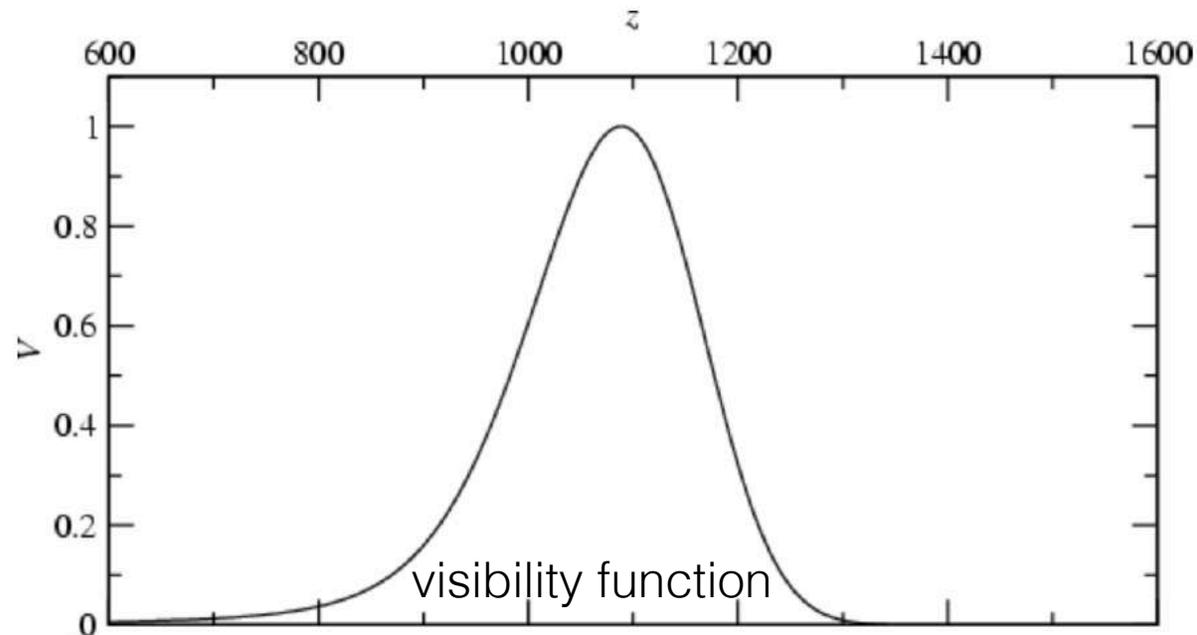
Note: This applies also on scales larger than the sound horizon. On larger scales, S-W traces potential fluctuations unaffected by sound waves.

Silk (photon diffusion) damping, effect #1

We calculated $z_{LS} \sim 1,100$ by setting $\tau = 1$

In reality, scattering is probabilistic and there is a probability distribution of last scattering redshift, called the visibility function

Temperature fluctuations on scales $\ll \Delta l_s$ average out along line of sight

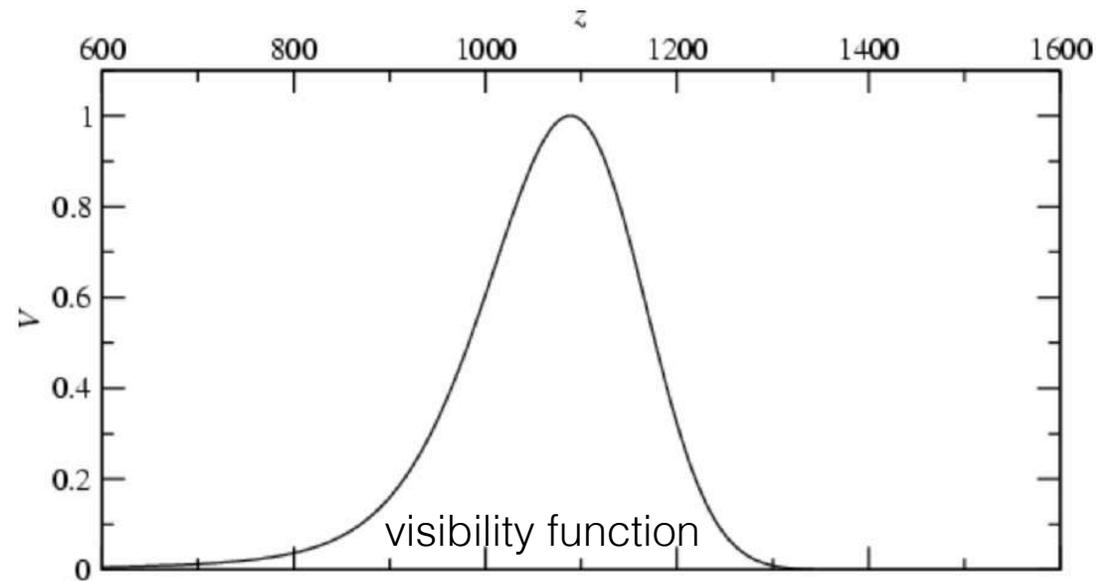


Silk (photon diffusion) damping, effect #2

Photons are not effectively dragged into potential wells of size $\ll \Delta l_{\text{LS}} \sim \lambda_{\text{T}}(\text{LS})$ (absent strong coupling with baryons, their tendency is to stream out at speed c)

But photons have significant $\rho_{\text{rad}} \sim \rho_{\text{b}} \sim (1/6)\rho_{\text{DM}}$, so they smooth out potential fluctuations $\delta\Phi$ on scales $\lesssim \lambda_{\text{T}}(\text{LS})$

Together, Silk damping effects #1 and #2 strongly suppress $\Delta T/T$ for $l \gtrsim 1,000$



Putting it all together: different CMB regimes

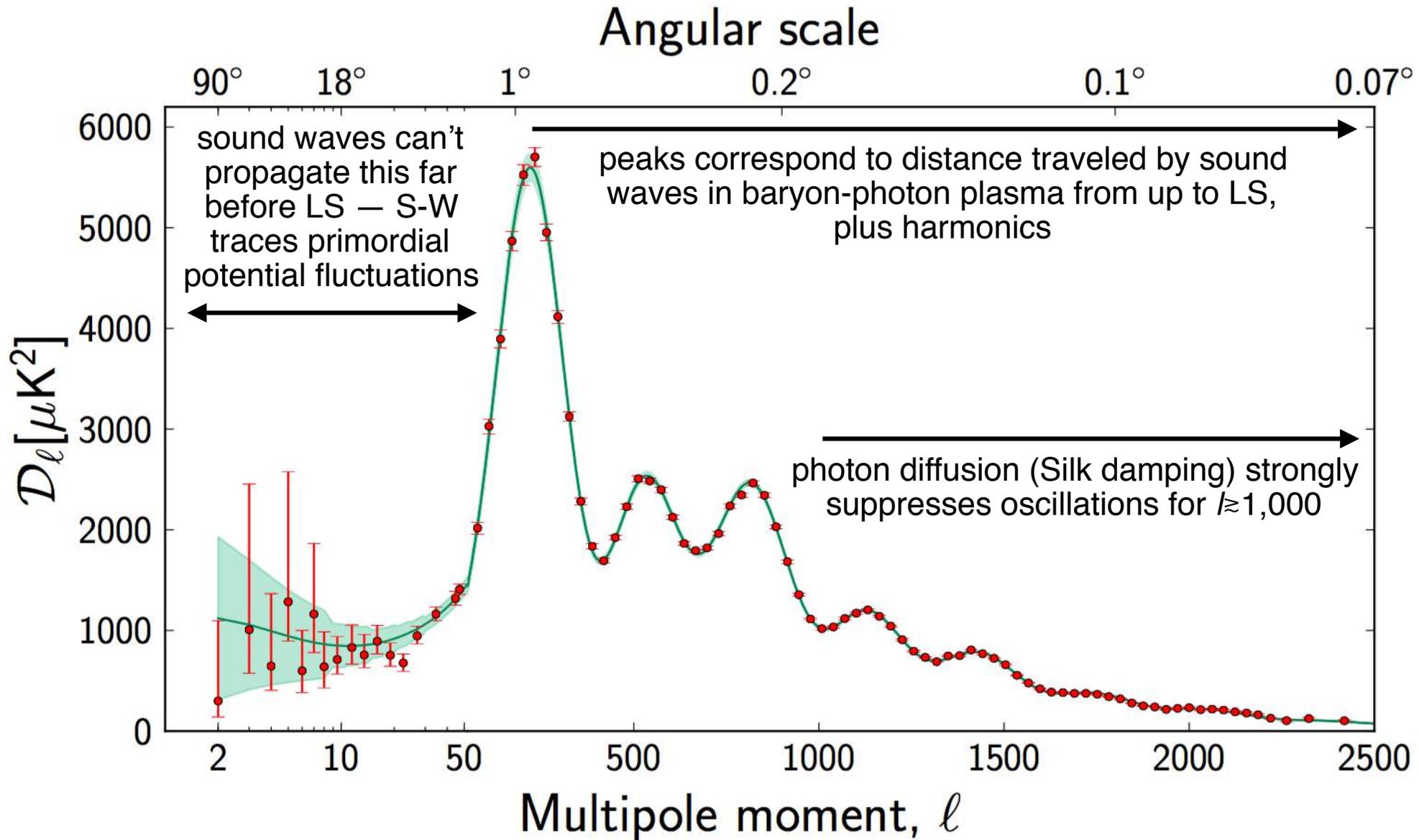
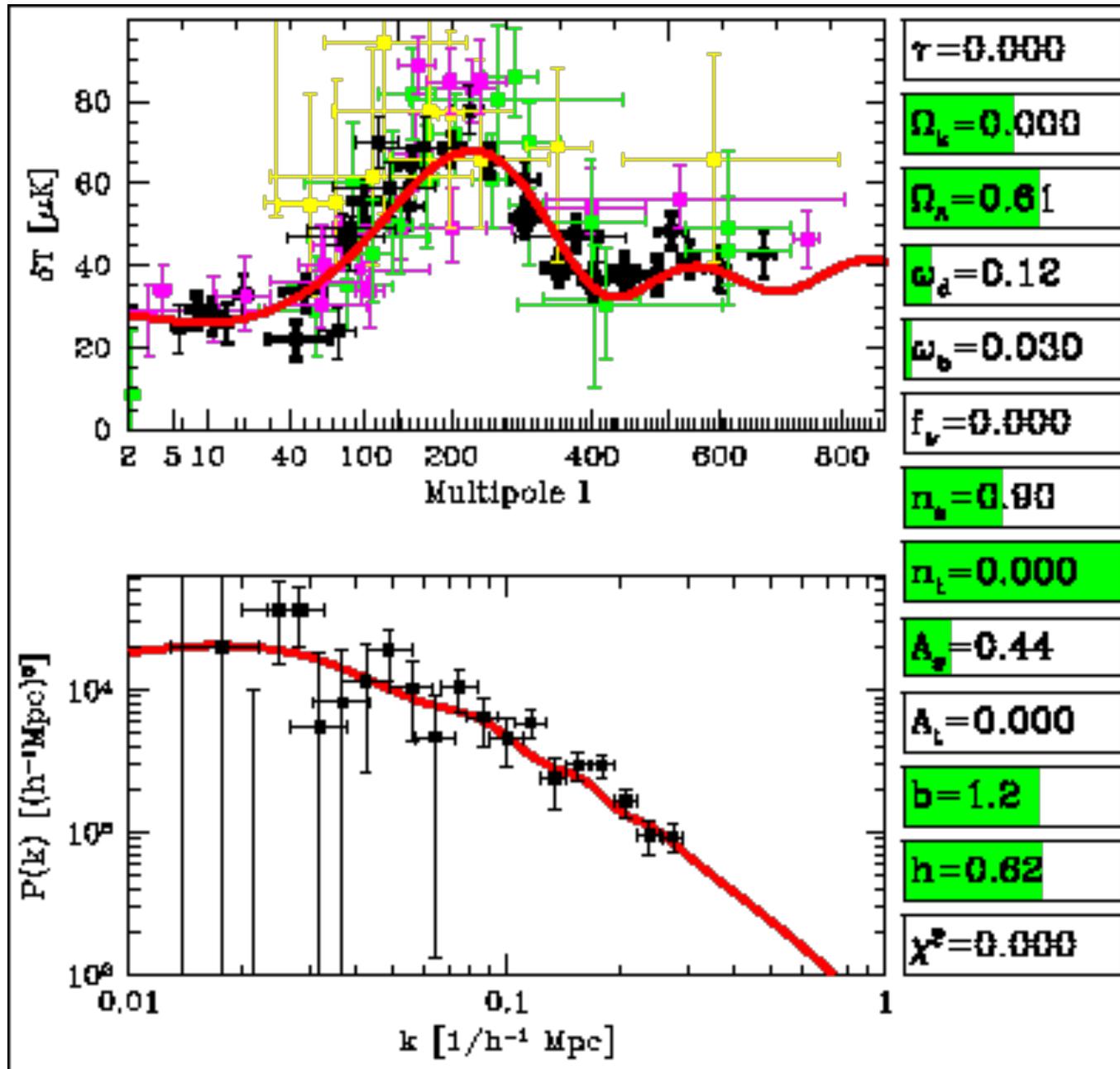


Figure 37. The 2013 *Planck* CMB temperature angular power spectrum. The error bars include cosmic variance, whose magnitude is indicated by the green shaded area around the best fit model. The low- ℓ values are plotted at 2, 3, 4, 5, 6, 7, 8, 9.5, 11.5, 13.5, 16, 19, 22.5, 27, 34.5, and 44.5.

Cosmological parameter estimation from the CMB power spectrum

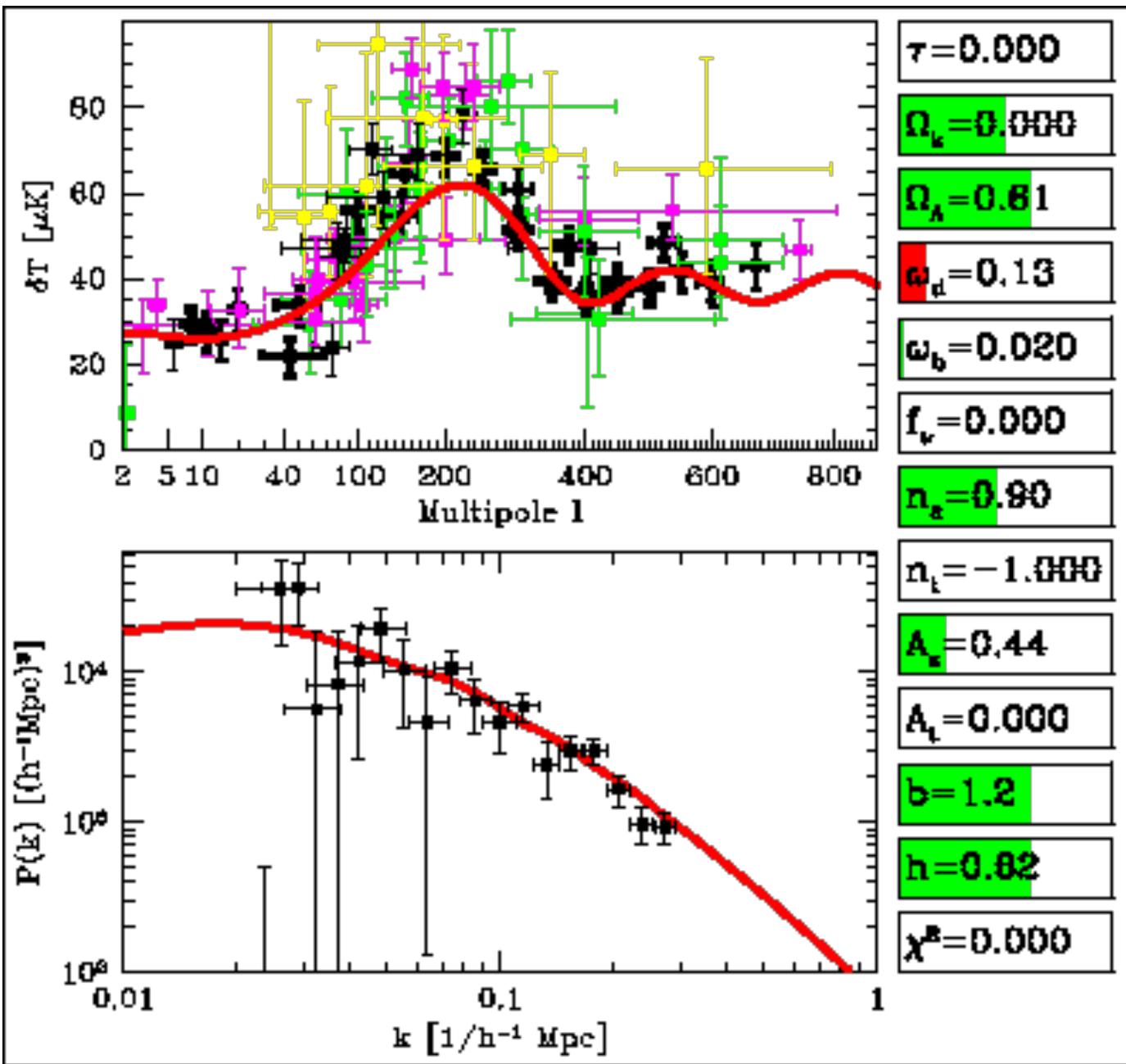
Varying baryon fraction



$$\omega_x \equiv \Omega_x h^2$$

more baryons \rightarrow higher peaks

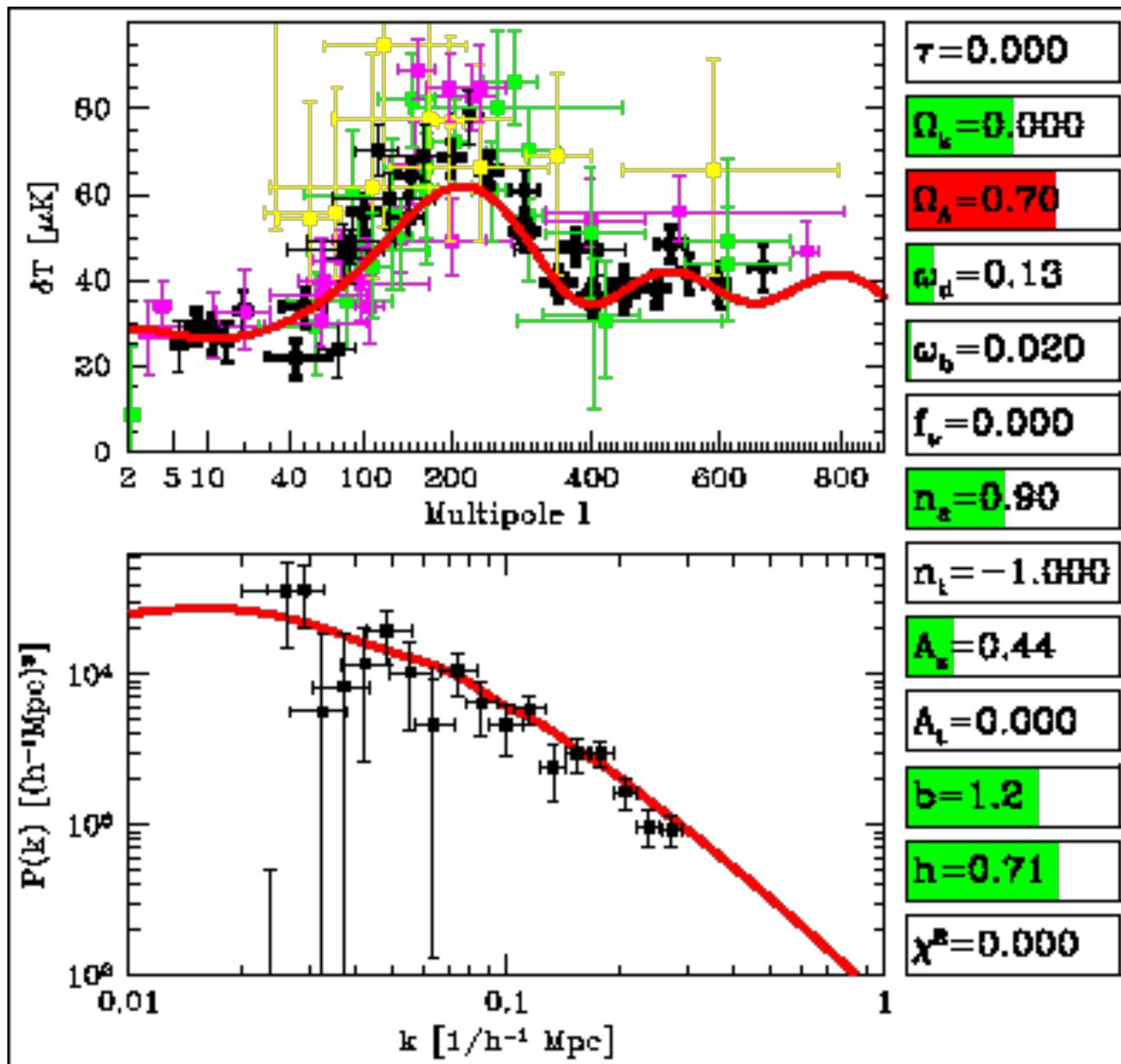
Varying dark matter mass density



$$\omega_x \equiv \Omega_x h^2$$

more dark matter \rightarrow lower peaks

Varying cosmological constant



$$\omega_x \equiv \Omega_x h^2$$

Λ dynamically unimportant at LS but changes $d_{\text{diam}}(z_{\text{LS}})$